Kensington Community Primary School's

Calculation Policy

Contents

Overview

Mastery Approach

At the centre of the mastery approach to the teaching of mathematics is the belief that **all children have the potential to succeed**. They should have access to the same curriculum content and, rather than being extended with new learning, they **should deepen their conceptual understanding** by tackling challenging and varied problems. Similarly, with calculation strategies, children must not simply rote learn procedures but demonstrate their understanding of these procedures using concrete materials and pictorial representations. This policy outlines the different calculation strategies that should be taught and used in Year 1 to Year 6 in line with the requirements of the 2014 Primary National Curriculum.

How To Use The Policy

This mathematics policy is a guide for all staff at Kensington Community Primary school and has been adapted from work by the NCETM. It is purposely set out as a progression of mathematical skills and not into year group phases to encourage a flexible approach to teaching and learning. It is expected that teachers will use their professional judgement as to when consolidation of existing skills is required or if to move onto the next concept. However, the focus must always remain on breadth and depth rather than accelerating through concepts. Children should not be extended with new learning before they are ready, they should deepen their conceptual understanding by tackling challenging and varied problems. All teachers have been given schemes of work adapted from the White Rose Maths Hub and are required to base their planning around their year groups modules and not to move onto a higher year groups scheme work. These modules are affiliated to the workings of the 2014 Maths Programme of Study.

Teachers can use any teaching resources that they wish to use, and the policy does not recommend one set of resources over another, rather that, a variety of resources are used. For each of the four rules of number, different strategies are laid out, together with examples of what concrete materials can be used and how, along with suggested pictorial representations. The principle of the concrete-pictorial-abstract (CPA) approach [Make it, Draw it, Write it] is for children to have a true understanding of a mathematical concept, they need to master all three phases within a year group's scheme of work.

Mental Strategies

Addition and Subtraction

	Recall	Mental Calculation Skills	Mental Methods or Strategies
Year 1	• Number pairs with a total of 10, e.g. 3 + 7, or what to add to a single-digit number to make 10, e.g. 3 + □ = 10 • Addition facts for totals to at least 5, e.g. 2 + 3, 4 + 3 • Addition doubles for all numbers to at least 10, e.g. 8 + 8	• Add or subtract a pair of single-digit numbers, e.g. 4 + 5, 8 – 3 • Add or subtract a single digit number to or from a teens number, e.g. 13 + 5, 17 – 3 • Add or subtract a single digit to or from 10, and add a multiple of 10 to a single digit number, e.g. 10 + 7, 7 + 30 • Add near doubles, e.g. 6 + 7	• Reorder numbers when adding, e.g. put the larger number first • Count on or back in ones, twos or tens • Partition small numbers, e.g. 8 + 3 = 8 + 2 + 1 • Partition and combine tens and ones • Partition: double and adjust, e.g. 5 + 6 = 5 + 5 + 1
Year 2	• Addition and subtraction facts for all numbers up to at least 10, e.g. 3 + 4, 8 – 5 • Number pairs with totals to 20 • All pairs of multiples of 10 with totals up to 100, e.g. 30 + 70, or 60 + □ = 100 • What must be added to any two-digit number to make the next multiple of 10, e.g. 52 + □ = 60 • Addition doubles for all numbers to 20, e.g. 17 + 17 and multiples of 10 to 50, e.g. 40 + 40	• Add or subtract a pair of single-digit numbers, including crossing 10, e.g. 5 + 8, 12 – 7 • Add any single-digit number to or from a multiple of 10, e.g. 60 + 5 • Subtract any single-digit number from a multiple of 10, e.g. 80 – 7 • Add or subtract a single digit number to or from a two-digit number, including crossing the tens boundary, e.g. 23 + 5, 57 – 3, then 28 + 5, 52 – 7 • Add or subtract a multiple of 10 to or from any two-digit number, e.g. 27 + 60, 72 – 50 • Add 9, 19, 29, … or 11, 21, 31, … • Add near doubles, e.g. 13 + 14, 39 + 40	• Reorder numbers when adding • Partition: bridge through 10 and multiples of 10 when adding and subtracting • Partition and combine multiples of tens and ones • Use knowledge of pairs making 10 • Partition: count on in tens and ones to find the total • Partition: count on or back in tens and ones to find the difference • Partition: add a multiple of 10 and adjust by 1 • Partition: double and adjust

Year 3	• Addition and subtraction facts for all numbers to 20, e.g. 9 + 8, 17 – 9, drawing on knowledge of inverse operations • Sums and differences of multiples of 10, e.g. 50 + 80, 120 – 90 • Pairs of two-digit numbers with a total of 100, e.g. 32 + 68, or 32 + □ = 100 • Addition doubles for multiples of 10 to 100, e.g. 90 + 90	• Add and subtract groups of small numbers, e.g. 5 – 3 + 2 • Add or subtract a two-digit number to or from a multiple of 10, e.g. 50 + 38, 90 – 27 • Add and subtract two-digit numbers e.g. 34 + 65, 68 – 35 • Add near doubles, e.g. 18 + 16, 60 + 70	• Reorder numbers when adding • Identify pairs totalling 10 or multiples of 10 • Partition: add tens and ones separately, then recombine • Partition: count on in tens and ones to find the total • Partition: count on or back in tens and ones to find the difference • Partition: add or subtract 10 or 20 and adjust • Partition: double and adjust • Partition: count on or back in minutes and hours, bridging through 60 (analogue times)
Year 4	• Sums and differences of pairs of multiples of 10, 100 or 1000 • Addition doubles of numbers 1 to 100, e.g. 38 + 38, and the corresponding halves • What must be added to any three-digit number to make the next multiple of 100, e.g. 521 + □ = 600 • Pairs of fractions that total 1	• Add or subtract any pair of two-digit numbers, including crossing the tens and 100 boundary, e.g. 47 + 58, 91 – 35 • Add or subtract a near multiple of 10, e.g. 56 + 29, 86 – 38 • Add near doubles of two-digit numbers, e.g. 38 + 37 • Add or subtract two-digit or three-digit multiples of 10, e.g. 120 – 40, 140 + 150, 370 – 180	• Count on or back in hundreds, tens and ones • Partition: add tens and ones separately, then recombine • Partition: subtract tens and then ones, e.g. subtracting 27 by subtracting 20 then 7 • Subtract by counting up from the smaller to the larger number • Partition: add or subtract a multiple of 10 and adjust, e.g. 56 + 29 = 56 + 30 – 1, or 86 – 38 = 86 – 40 + 2 • Use knowledge of place value and related calculations, e.g. work out 140 + 150 = 290 using 14 + 15 = 29 • Partition: count on or back in minutes and hours, bridging through 60 (analogue and digital)

Year 5	• Sums and differences of decimals, e.g. 6.5 + 2.7, 7.8 – 1.3 • Doubles and halves of decimals, e.g. half of 5.6, double 3.4 • What must be added to any four-digit number to make the next multiple of 1000, e.g. 4087 + □ = 5000 • What must be added to a decimal with units and tenths to make the next whole number, e.g. 7.2 + □ = 8	• Add or subtract a pair of two-digit numbers or three-digit multiples of 10, e.g. 38 + 86, 620 – 380, 350+ 360 • Add or subtract a near multiple of 10 or 100 to any two-digit or three-digit number, e.g. 235 + 198 • Find the difference between near multiples of 100, e.g. 607 – 588, or of 1000, e.g. 6070 – 4087 • Add or subtract any pairs of decimal fractions each with units and tenths, e.g. 5.7 + 2.5, 6.3 – 4.	• Count on or back in hundreds, tens, ones and tenths • Partition: add hundreds, tens or ones separately, then recombine • Subtract by counting up from the smaller to the larger number • Add or subtract a multiple of 10 or 100 and adjust • Partition: double and adjust • Use knowledge of place value and related calculations, e.g. 6.3 – 4.8 using 63 – 48 • Partition: count on or back in minutes and hours, bridging through 60 (analogue and digital times)
Year 6	• Addition and subtraction facts for multiples of 10 to 1000 and decimal numbers with one decimal place, e.g. 650 + □ = 930 or □ – 1.4 = 2.5 • What must be added to a decimal with units, tenths and hundredths to make the next whole number, e.g. 7.26 + □ = 8	• Add or subtract pairs of decimals with units, tenths or hundredths, e.g. 0.7 + 3.38 • Find doubles of decimals each with units and tenths, e.g. 1.6 + 1.6 • Add near doubles of decimals, e.g. 2.5 + 2.6 • Add or subtract a decimal with units and tenths, that is nearly a whole number, e.g. 4.3 + 2.9, 6.5 – 3.8	• Count on or back in hundreds, tens, ones, tenths and hundredths • Use knowledge of place value and related calculations, e.g. 680 + 430, 6.8 + 4.3, 0.68 + 0.43 can all be worked out using the related calculation 68 + 43 • Use knowledge of place value and of doubles of two-digit whole numbers • Partition: double and adjust • Partition: add or subtract a whole number and adjust, e.g. 4.3 + 2.9 = 4.3 + 3 – 0.1, 6.5 – 3.8 = 6.5 – 4 + 0.2 • Partition: count on or back in minutes and hours, bridging through 60 (analogue and digital)

Multiplication and Division

	Recall	Mental Calculation Skills	Mental Methods or Strategies
Year 1	• Doubles of all numbers to 10, e.g. double 6 • Odd and even numbers to 20	• Count on from and back to zero in ones, twos, fives or tens	• Use patterns of last digits, e.g. 0 and 5 when counting in fives
Year 2	• Doubles of all numbers to 20, e.g. double 13, and corresponding halves • Doubles of multiples of 10 to 50, e.g. double 40, and corresponding halves • Multiplication facts for the 2, 5 and 10 times-tables, and corresponding division facts • Add odd and even numbers to 100	• Double any multiple of 5 up to 50, e.g. double 35 • Halve any multiple of 10 up to 100, e.g. halve 90 • Find half of even numbers to 40 • Find the total number of objects when they are organised into groups of 2, 5 or 10	• Partition: double the tens and ones separately, then recombine • Use knowledge that halving is the inverse of doubling and that doubling is equivalent to multiplying by two • Use knowledge of multiplication facts from the 2, 5 and 10 times-tables, e.g. recognise that there are 15 objects altogether because there are three groups of five
Year 3	• Multiplication facts for the 2, 3, 4, 5, 6 and 10 times tables, and corresponding division facts • Doubles of multiples of 10 to 100, e.g. double 90, and corresponding halves	• Double any multiple of 5 up to 100, e.g. double 35 • Halve any multiple of 10 up to 200, e.g. halve 170 • Multiply one-digit or two-digit numbers by 10 or 100, e.g. 7 × 100, 46 × 10, 54 x 100 • Find unit fractions of numbers and quantities involving halves, thirds, quarters, fifths and tenths	• Partition: when doubling, double the tens and ones separately, then recombine • Partition: when halving, halve the tens and ones separately, then recombine • Use knowledge that halving and doubling are inverse operations • Recognise that finding a unit fraction is equivalent to dividing by the denominator and use knowledge of division facts • Recognise that when multiplying by 10 or 100 the digits move one or two places to the left and zero is used as a place holder

Year 4	• Multiplication facts to 10 × 10 and the corresponding division facts • Doubles of numbers 1 to 100, e.g. double 58, and corresponding halves • Doubles of multiples of 10 and 100 and corresponding halves • Fraction and decimal equivalents of one-half, quarters, tenths and hundredths, e.g. 3 tenths is 0.3 and 3 hundredths is 0.03 • Factor pairs for known multiplication facts	• Double any two-digit number, e.g. double 39 • Double any multiple of 10 or 100, e.g. double 340, double 800, and halve the corresponding multiples of 10 and 100 • Halve any even number to 200 • Find unit fractions and simple non-unit fractions of numbers and quantities, e.g. 3/8 of 24 • • Multiply and divide numbers to 1000 by 10 and then 100 (whole-number answers), e.g. 325 × 10, 42 × 100, 120 ÷ 10, 600 ÷ 100, 850 ÷ 10 • Multiply a multiple of 10 to 100 by a single-digit number, e.g. 40 × 3 • Multiply numbers up to 20 by a single-digit, e.g. 17 × 3 • Identify the remainder when dividing by 2, 5 or 10 • Give the factor pair associated with a multiplication fact, e.g. identify that if 2 x 3 = 6 then 6 has the factor pair 2 and 3	• Partition: double or halve the tens and ones separately, then recombine • Use understanding that when a number is multiplied or divided by 10 or 100, its digits move one or two places to the left or the right and zero is used as a place holder • Use knowledge of multiplication facts and place value, e.g. 7 x 8 = 56 to find 70 x 8, 7 x 80 • Use partitioning and the distributive law to multiply, e.g. 13 × 4 = (10 + 3) × 4 = (10 × 4) + (3 × 4) = 40 + 12 = 52

Year 5	• Squares to 10 × 10 • Division facts corresponding to tables up to 10 × 10, and the related unit fractions, e.g. 7 × 9 = 63 so one-ninth of 63 is 7 and one-seventh of 63 is 9 • Percentage equivalents of one-half, one-quarter, three-quarters, tenths and hundredths • Factor pairs to 100	• Multiply and divide two-digit numbers by 4 or 8, e.g. 26 × 4, 96 ÷ 8 • Multiply two-digit numbers by 5 or 20, e.g. 320 × 5, 14 × 20 • Multiply by 25 or 50, e.g. 48 × 25, 32 × 50 • Double three-digit multiples of 10 to 500, e.g. 380 × 2, and find the corresponding halves, e.g. 760 ÷ 2 • Find the remainder after dividing a two-digit number by a single-digit number, e.g. 27 ÷ 4 = 6 R 3 • Multiply and divide whole numbers and decimals by 10, 100 or 1000, e.g. 4.3 × 10, 0.75 × 100, 25 ÷ 10, 673 ÷ 100, 74 ÷ 100 • Multiply pairs of multiples of 10, e.g. 60 × 30, and a multiple of 100 by a single digit number, e.g. 900 × 8 • Divide a multiple of 10 by a single-digit number (whole number answers) e.g. 80 ÷ 4, 270 ÷ 3 • Find fractions of whole numbers or quantities, e.g. 2/3 of 27, 4/5 of 70 kg • Find 50%, 25% or 10% of whole numbers or quantities, e.g. 25% of 20 kg, 10% of £80 • Find factor pairs for numbers to 100, e.g. 30 has the factor pairs 1 × 30, 2 × 15, 3 × 10 and 5 × 6	• Multiply or divide by 4 or 8 by repeated doubling or halving • Form an equivalent calculation, e.g. to multiply by 5, multiply by 10, then halve; to multiply by 20, double, then multiply by 10 • Use knowledge of doubles/halves and understanding of place value, e.g. when multiplying by 50 multiply by 100 and divide by 2 • Use knowledge of division facts, e.g. when carrying out a division to find a remainder • Use understanding that when a number is multiplied or divided by 10 or 100, its digits move one or two places to the left or the right relative to the decimal point, and zero is used as a place holder • Use knowledge of multiplication and division facts and understanding of place value, e.g. when calculating with multiples of 10 • Use knowledge of equivalence between fractions and percentages, e.g. to find 50%, 25% and 10% • Use knowledge of multiplication and division facts to find factor pairs

Year 6	• Squares to 12 × 12 • Squares of the corresponding multiples of 10 • Prime numbers less than 100 • Equivalent fractions, decimals and percentages for hundredths, e.g. 35% is equivalent to 0.35 or $\frac{35}{100}$	• Multiply pairs of two-digit and single-digit numbers, e.g. 28 × 3 • Divide a two-digit number by a single-digit number, e.g. 68 ÷ 4 • Divide by 25 or 50, e.g. 480 ÷ 25, 3200 ÷ 50 • Double decimals with units and tenths, e.g. double 7.6, and find the corresponding halves, e.g. half of 15.2 • Multiply pairs of multiples of 10 and 100, e.g. 50 × 30, 600 × 20 • Divide multiples of 100 by a multiple of 10 or 100 (whole number answers), e.g. 600 ÷ 20, 800 ÷ 400, 2100 ÷ 300 • Multiply and divide two-digit decimals such as 0.8 × 7, 4.8 ÷ 6 • Find 10% or multiples of 10%, of whole numbers and quantities, e.g. 30% of 50 ml, 40% of £30, 70% of 200 g • Simplify fractions by cancelling • Scale up and down using known facts, e.g. given that three oranges cost 24p, find the cost of four oranges • Identify numbers with odd and even numbers of factors and no factor pairs other than 1 and themselves	• Partition: use partitioning and the distributive law to divide tens and ones separately, e.g. 92 ÷ 4 = (80 + 12) ÷ 4 = 20 + 3 = 23 • Form an equivalent calculation, e.g. to divide by 25, divide by 100, then multiply by 4; to divide by 50, divide by 100, then double • Use knowledge of the equivalence between fractions and percentages and the relationship between fractions and division • Recognise how to scale up or down using multiplication and division, e.g. if three oranges cost 24p: one orange costs 24 ÷ 3 = 8p four oranges cost 8 × 4 = 32p • Use knowledge of multiplication and division facts to identify factor pairs and numbers with only two factors

Written Methods

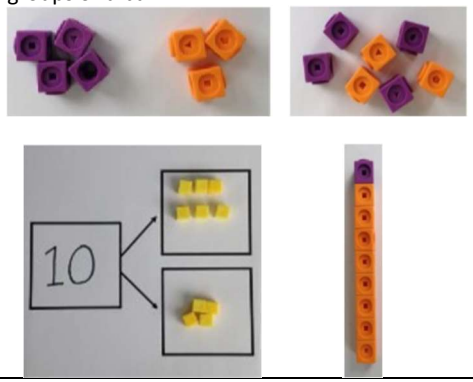
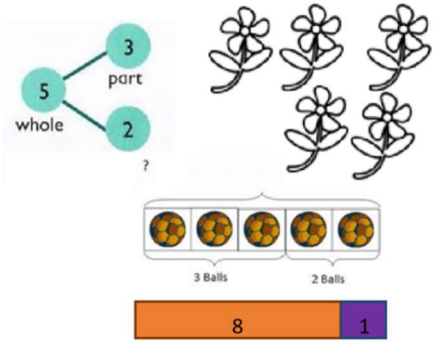
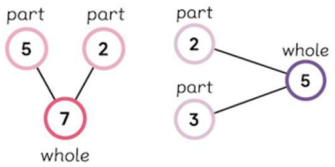
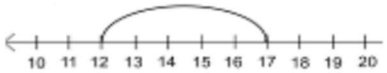
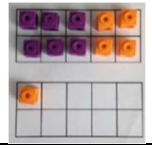
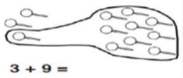
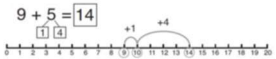
Overview of Objectives

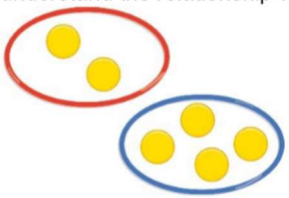
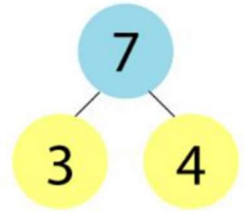
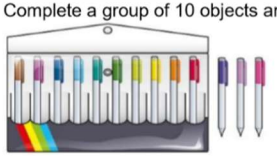
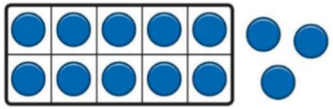
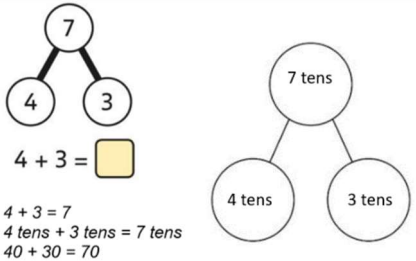
	EYFS/Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
Addition	Combining two parts to make a whole: part whole model. Starting at the bigger number and counting on- using cubes. Regrouping to make 10 using ten frame.	Adding three single digits. Use of base 10 to combine two numbers.	Column method- regrouping. Using place value counters (up to 3 digits).	Column method- regrouping. (up to 4 digits)	Column method- regrouping. Use of place value counters for adding decimals.	Column method- regrouping. Abstract methods. Place value counters to be used for adding decimal numbers.
Subtraction	Taking away ones Counting back Find the difference Part whole model Make 10 using the ten frame	Counting back Find the difference Part whole model Make 10 Use of base 10	Column method with regrouping. (up to 3 digits using place value counters)	Column method with regrouping. (up to 4 digits)	Column method with regrouping. Abstract for whole numbers. Start with place value counters for decimals- with the same amount of decimal places.	Column method with regrouping. Abstract methods. Place value counters for decimals- with different amounts of decimal places.

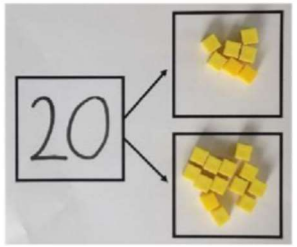
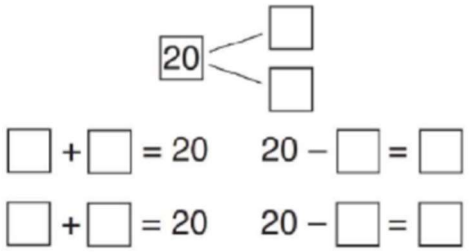
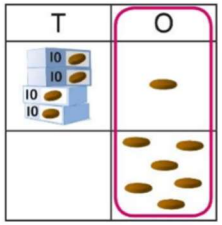
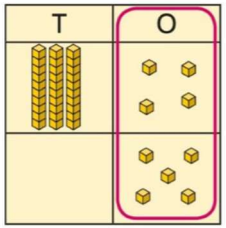
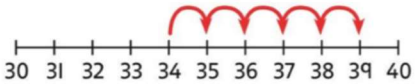
	EYFS/Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
Multiplication	Recognising and making equal groups. Doubling Counting in multiples Use cubes, Numicon and other objects in the classroom	Arrays- showing commutative multiplication	Arrays 2d × 1d using base 10	Column multiplication- introduced with place value counters. (2 and 3 digit multiplied by 1 digit)	Column multiplication Abstract only but might need a repeat of year 4 first(up to 4 digit numbers multiplied by 1 or 2 digits)	Column multiplication Abstract methods (multi-digit up to 4 digits by a 2 digit number)
Division	Sharing objects into groups Division as grouping e.g. I have 12 sweets and put them in groups of 3, how many groups? Use cubes and draw round 3 cubes at a time.	Division as grouping Division within arrays- linking to multiplication Repeated subtraction	Division with a remainder-using lollipop sticks, times tables facts and repeated subtraction. 2d divided by 1d using base 10 or place value counters	Division with a remainder Short division (up to 3 digits by 1 digit- concrete and pictorial)	Short division (up to 4 digits by a 1 digit number including remainders)	Short division Long division with place value counters (up to 4 digits by a 2 digit number) Children should exchange into the tenths and hundredths column too

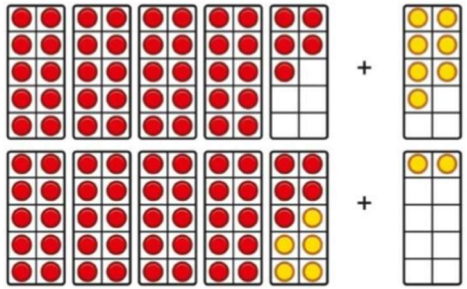
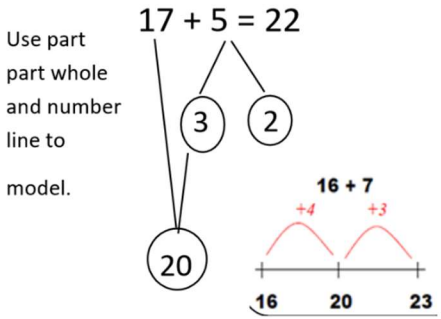
Addition

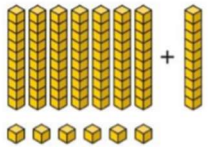
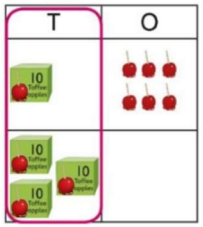
Key language: sum, total, parts and wholes, plus, add, altogether, more, 'is equal to' 'is the same as'.

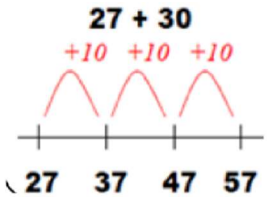
Objective	Concrete	Pictorial	Abstract
Combine two parts to make a whole: part-whole model.	Use a concrete part-part-whole model, either as a groups or a bar. 	Use pictures to add as a group or as a bar. 	Use values in the part-part whole model. 
Starting at the bigger number and counting on.	Start with the larger number on the bead string and then count on to the smaller number 1 by 1 to find the answer. 	Start at the larger number on the number line and count on in ones or in one jump to find the answer. 	Place the larger number in your head and count on the smaller number to find your answer.
Regrouping to make 10. (A key skill for the column method)	Start with the bigger number and use the smaller number to make 10. Use ten frames.  	Use pictures or a number line. Regroup or partition the smaller number using the part-part whole model.  	A mental approach enables the children to partition the second value to make 10 and then count on the remaining amount. *How many more do I need to make 10?* *How many do I still need to add on?*

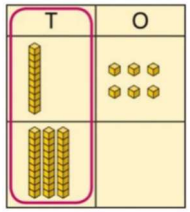
Represent and use number bonds and related subtraction facts within 20.	Sort people and objects into parts and understand the relationship with the whole.  *The parts are 2 and 4. The whole is 6.*	Children draw to represent the parts and understand the relationship with the whole.  *The parts are 1 and 5. The whole is 6.*	 Represents these fact families 3 + 4 = 7 4 + 3 = 7
To recognise teen numbers as a complete 10 and some more.	Complete a group of 10 objects and count more.  *13 is 10 and 3 more.*	Use a ten frame to support understanding of a complete 10 for teen numbers.  *13 is 10 and 3 more.*	1 ten and 3 ones is equal to 13. 10 + 3 = 13
To add multiples of 10.	Model using dienes and bead strings. Use known bonds and unitising to add 10s.  *I know that 4 + 3 = 7.* *So, I know that 4 tens add 3 tens is 7 tens.*	Use known bonds and unitising to add 10's  I know that 4 + 3 = 7 So I know, 4 tens add 3 tens is 7 tens.	 *4 + 3 = 7* *4 tens + 3 tens = 7 tens* *40 + 30 = 70*

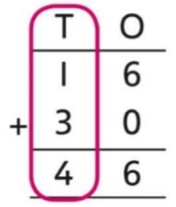
Use known number facts. (Part-part-whole)	 Children explore ways of making numbers within 20	 	Explore commutativity of addition by swapping the addends to build a fact family. Explore the concept of the inverse relationship of addition and subtractions and use this to check calculations. □ + 1 = 16 16 − 1 = □ 1 + □ = 16 16 − □ = 1
Add two-digit numbers and ones. (Not bridging 10)	Add the 1s to find the total. Use known bonds within 10.  *41 is 4 tens and 1 one.* This can also be done in a place value grid.  	Add the 1s.  *34 is 3 tens and 4 ones.* *4 ones and 5 ones are 9 ones.* *The total is 3 tens and 9 ones.*  	Understand the link between counting on and using known number facts. Children should be encouraged to use known number bonds to improve efficiency and accuracy.  This can be represented horizontally or vertically. *34 + 5 = 39* *or* T O 3 4 + 5 9

<table>
<tr>
<td>Add 2-digit numbers and ones.
(Bridging 10)</td>
<td>Complete using counters and tens frames
  </td>
<td>Use part-whole model and number lines to model.
Use part part whole and number line to model.
 
</td>
<td>Explore related facts
17 + 5 = 22
 

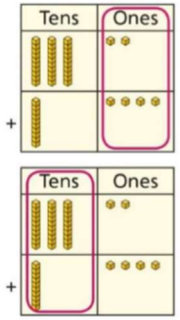
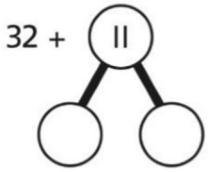
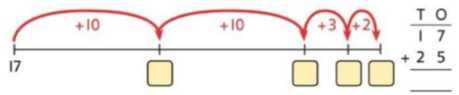
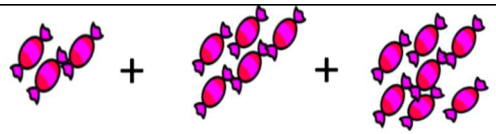
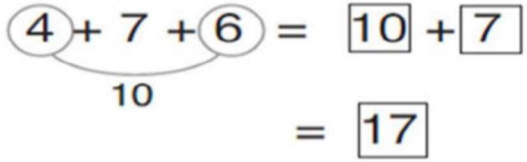
17 + 5 = 22
5 + 17 = 22
22 – 17 = 5
22 – 5 = 17</td>
</tr>
<tr>
<td>Add two-digit numbers and tens.</td>
<td> 
66 is 6 tens and 6 ones.
66 + 10 = 76
 

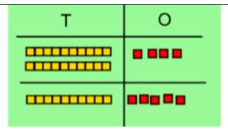
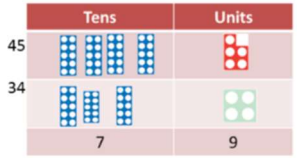
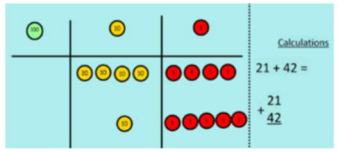
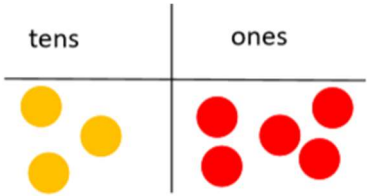
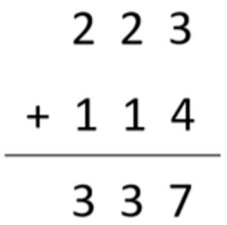
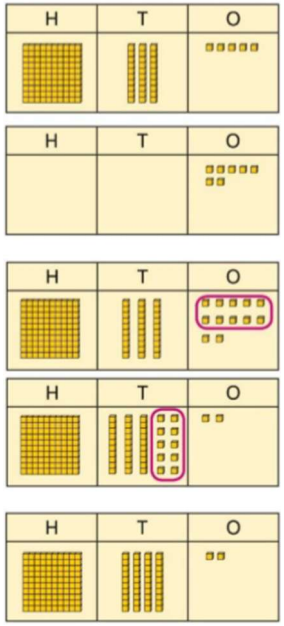
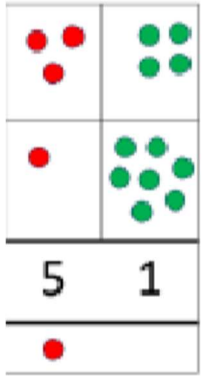
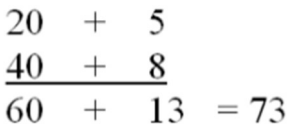
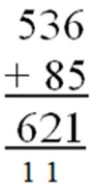
16 is 1 ten and 6 ones.
30 is 3 tens.
There are 4 tens and 6 ones in total.</td>
<td> 

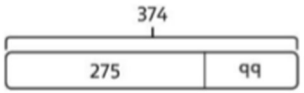
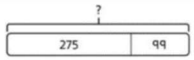
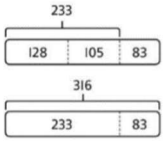
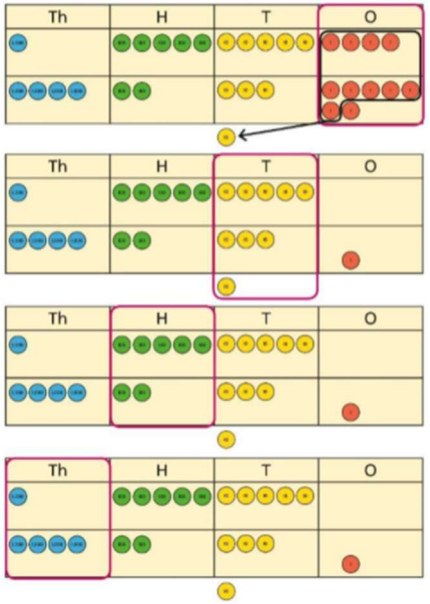
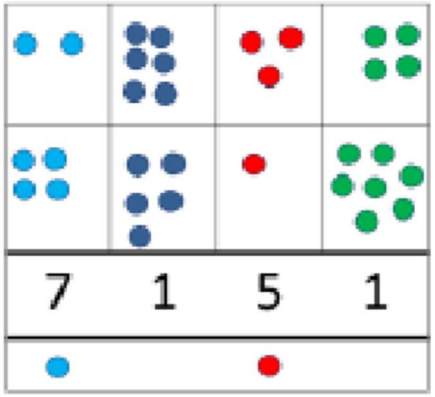
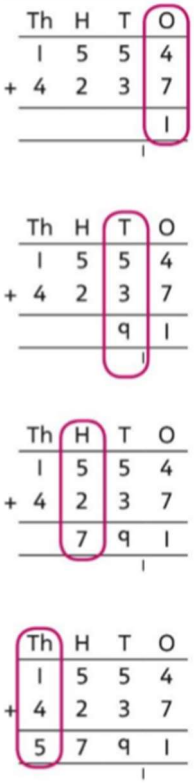
 

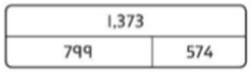
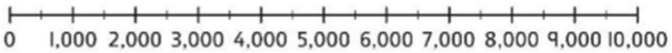
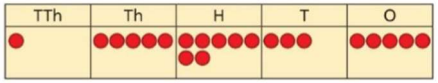
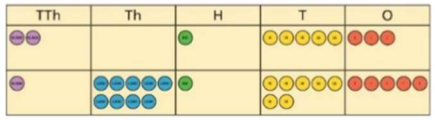
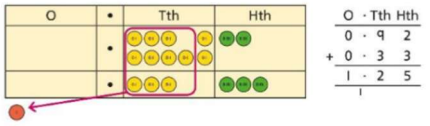
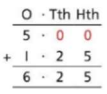
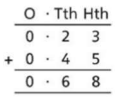
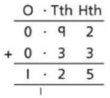
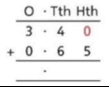
16 is 1 ten and 6 ones.
30 is 3 tens.
There are 4 tens and 6 ones in total.</td>
<td>37 + 20 = ?
30 + 20 = 50
50 + 7 = 57
37 + 20 = 57
 

1 + 3 = 4
1 ten + 3 tens = 4 tens
16 + 30 = 46</td>
</tr>
</table>

Add 2-digit numbers.	Add the 10s and 1s separately.  5 + 3 = 8 There are 8 ones in total. 3 + 2 = 5 There are 5 tens in total. 35 + 23 = 58 Progress onto adding the ones and then the tens. 	Use a part-whole model to support.  *11 = 10 + 1* *32 + 10 = 42* *42 + 1 = 43* *32 + 11 = 43*	Add the 10s and the 1s separately, bridging 10s where required. A number line can support the calculations.  *17 + 25* T O 3 2 + 1 4 6 T O 3 2 + 1 4 4 6
Add three 1-digit numbers.	 Combine to make 10 first if possible, or bridge 1o then add third digit	 Regroup and draw representation. 	 Combine the two numbers that make/ bridge ten then add on the third.

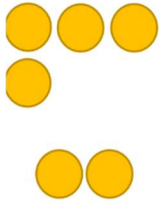
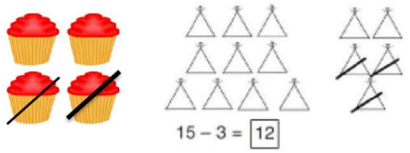
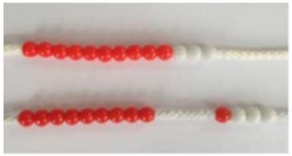
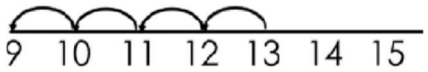
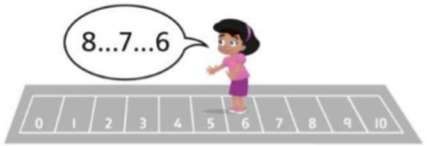
Column addition – no regrouping. (up to 3-digit numbers)	 Model using Dienes or nu-micon Add together the ones first, then the tens.  	Children move to drawing the counters using a tens and one frame. 	Add the ones first, then the tens, then the hundreds. 
Column addition with regrouping.	Children should explore this using unitised objects or physical apparatus. 	Children can draw a representation of the grid to further support their understanding, carrying the ten underneath the line. 	Start by partitioning the numbers before formal column to show the exchange. Expanded column addition  Formal column addition 

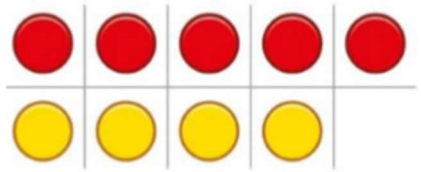
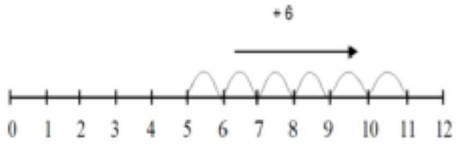
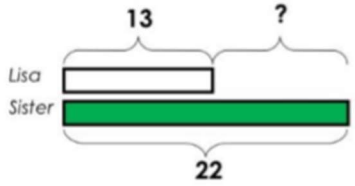
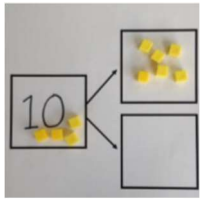
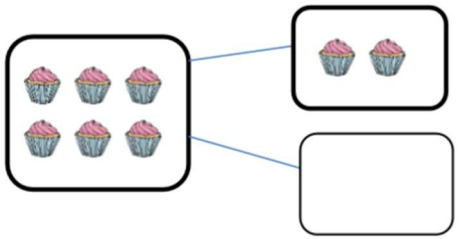
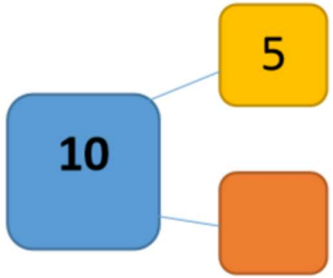
Representing addition problems, and selecting appropriate methods	Encourage children to use their own drawings and choices of place value equipment to represent problems with one or more steps. These representations will help them to select appropriate methods	Children understand and create bar models to represent addition problems. 275 + 99 = ?  275 + 99 = 37	 *I will add 100, then subtract 1 to find the solution.* *128 + 105 + 83 = ?* *I need to add three numbers.* 128 + 105 = 233 
Column addition with 4-digit numbers	Use place value equipment to model required exchanges.  Include examples that exchange in more than one column.	Use jottings alongside a place value grid to represent the value of each number in the calculation. 	

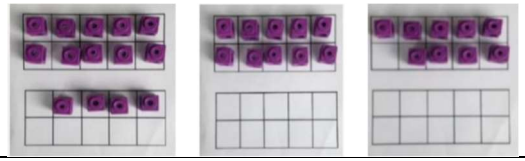
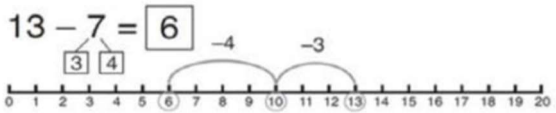
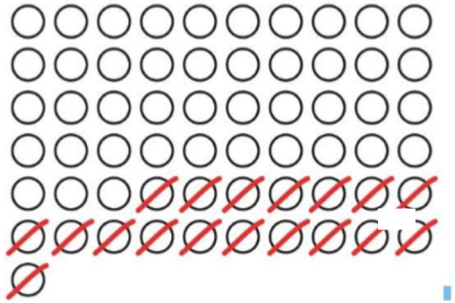
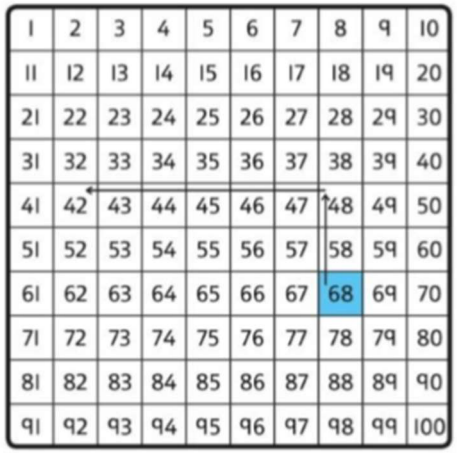
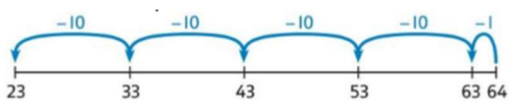
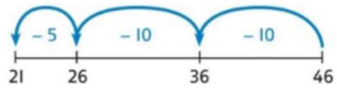
Representing additions and checking strategies		Bar models may be used to represent additions in problem contexts, and to justify mental methods where appropriate.   I chose to work out 574 + 800, then subtract 1.	Use rounding and estimating on a number line to check the reasonableness of an addition.  912 + 6,149 = ? I used rounding to work out that the answer should be approximately 1,000 + 6,000 = 7,000.
Column addition with several numbers of increasing complexity.	Use place value equipment to represent additions. Add a row of counters onto the place value grid to show 15,735 + 4,012. 	Represent additions, using place value equipment on a place value grid alongside written methods. 	Use column method, including exchanges. 
Adding decimals using column addition	Use place value equipment to represent additions. Show 0·23 + 0·45 using place value counters.  Include examples where the numbers of decimal places are different.  	Use jottings to represent the place value of each digit in the number.	Add using a column method, ensuring that children understand the link with place value.  Include exchange where required, alongside an understanding of place value.  Include additions where the numbers of decimal places are different. *3.4 + 0.65 = ?* 

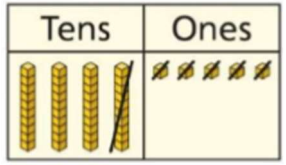
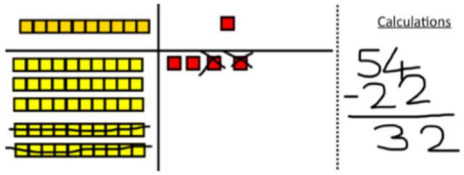
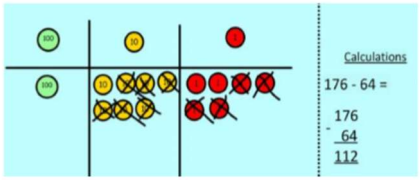
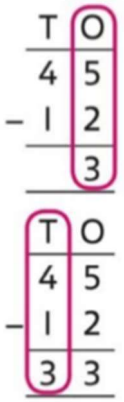
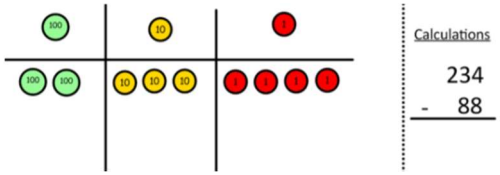
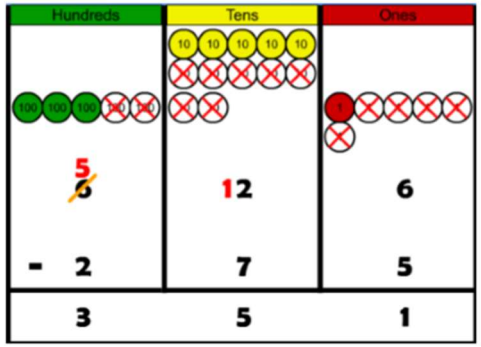
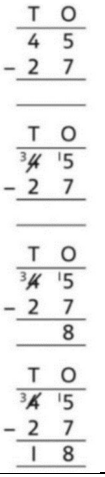
Subtraction

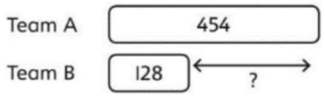
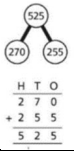
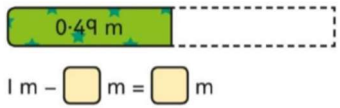
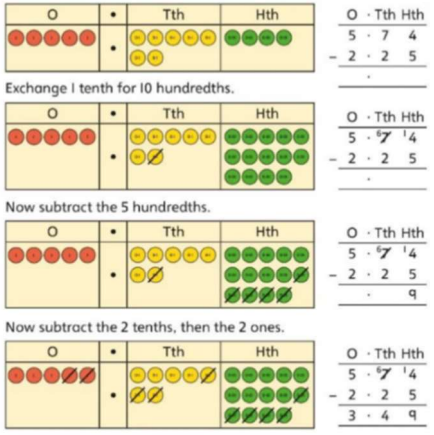
Key language: take away, less than, the difference, subtract, minus, fewer, decrease.

Objective	Concrete	Pictorial	Abstract
Taking away ones	Use physical objects, counters, cubes etc to show how objects can be taken away.  **6 – 2 = 4**	Cross out drawn objects to show what has been taken away. 	18 -3= 15 8 – 2 = 6
Counting back	Make the larger number in your subtraction. Move the beads along your bead string as you count backwards in ones. 13 – 4  Use counters and move them away from the group as you take them away counting backwards as you go. 	Count back on a number line or number track  Start at the bigger number and count back the smaller number showing the jumps on the number line.  This can progress all the way to counting back using two 2-digit numbers.	Put 13 in your head, count back 4. What number are you at? Use your fingers to help.  *9 – 3 = 6*

Find the difference	Arrange two groups so that the difference between the groups can be worked out.  8 is 2 more than 6. 6 is 2 less than 8. The difference between 8 and 6 is 2. 	Count on to find the difference.  Draw bars to find the difference between 2 numbers. *Lisa is 13 years old. Her sister is 22 years old. Find the difference in age between them.* 	Hannah has 23 sandwiches, Helen has 15 sandwiches. Find the difference between the number of sandwiches.
Part-part whole model	Link to addition- use the part whole model to help explain the inverse between addition and subtraction.  If 10 is the whole and 6 is one of the parts. What is the other part? 10 - 6 =	Use a pictorial representation of objects to show the part part whole model. 	Move to using numbers within the part whole model. 

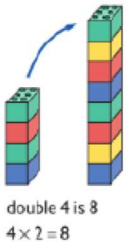
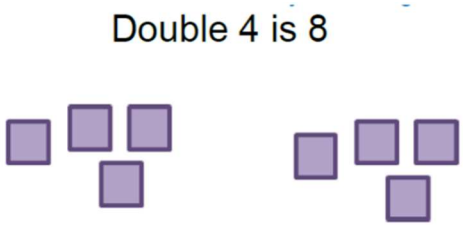
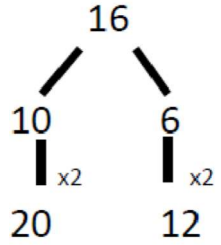
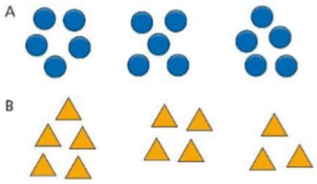
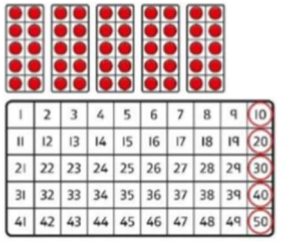
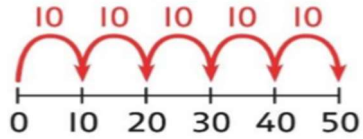
Bridging through 10 for subtraction	Make 14 on the ten frame. Take away the four first to make 10 and then takeaway one more so you have taken away 5. You are left with the answer of 9. 	Start at 13. Take away 3 to reach 10. Then take away the remaining 4 so you have taken away 7 altogether. You have reached your answer. 	16 – 8= How many do we take off to reach the 10? How many do we have left to take off?
Subtracting a 2-digit number	Subtract by taking away.  61 – 18 I took away 1 ten and 8 ones	Subtract the 10s and the 1s. This can be represented on a 100 square. 	Subtract the 10s and the 1s. This can be represented on a number line.  64 – 41 = ? 64 – 1 = 63 63 – 40 = 23 64 – 41 = 23  46 – 20 = 26 26 – 5 = 21 46 – 25 = 21

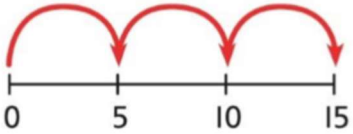
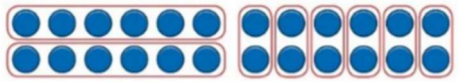
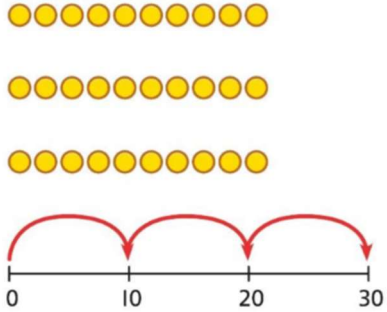
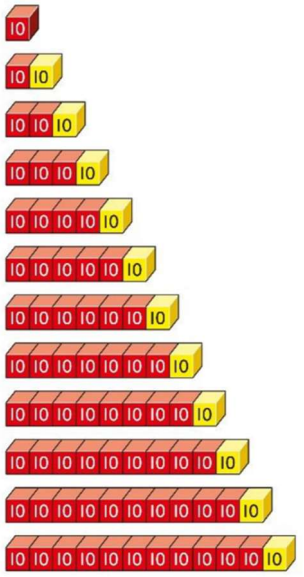
Column method without regrouping	Subtract the 1s. Then subtract the 10s. This may be done in or out of a place value grid. 	Draw the Base 10 or place value counters alongside the written calculation to help to show working.  	This will lead to a clear written column subtraction. 
Column method with regrouping	Use Base 10 to start with before moving on to place value counters. Start with one exchange before moving onto subtractions with 2 exchanges. Make the larger number with the place value counters. 	Draw the counters onto a place value grid and show what you have taken away by crossing the counters out as well as clearly showing the exchanges you make. 	Using column subtraction, exchange 1 ten for 10 ones. Then subtract the 1s. Then subtract the 10s. 

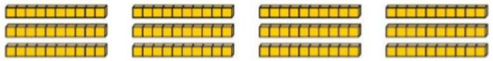
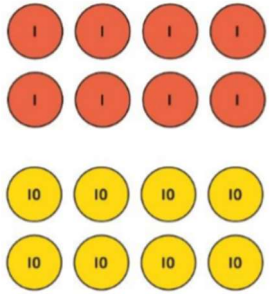
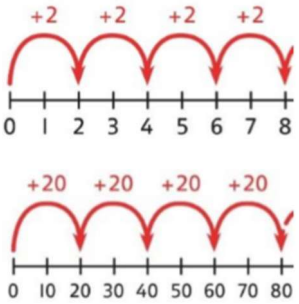
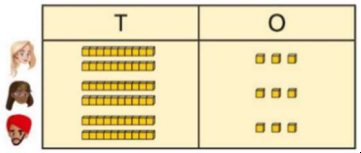
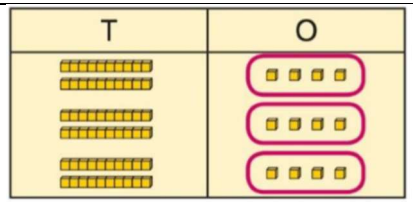
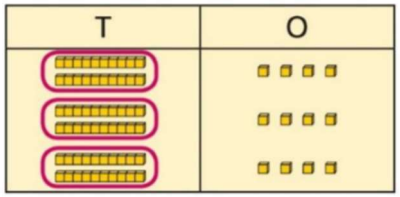
Representing subtraction problems		Use bar models to represent subtractions. 'Find the difference' is represented as two bars for comparison.  Bar models can also be used to show that a part must be taken away from the whole.	Children use alternative representations to check calculations and choose efficient methods. Children use inverse operations to check additions and subtractions. The part-whole model supports understanding. I have completed this subtraction. 525 − 270 = 255 I will check using addition. 
Subtracting decimals	Explore complements to a whole number by working in the context of length.  1 − 0·49 = ?  	Use a place value grid to represent the stages of column subtraction, including exchanges where required. The place value counters could be substituted for dots or circles to replicate the process.	Line up the digits appropriatley, ensuring the decimal point is consistently in the same place, including the answer. Use the 0 digit to make both values have the same number of decimal digits. O · Tth Hth Thth 3 · 9 2 1 − 3 · 7 5 0

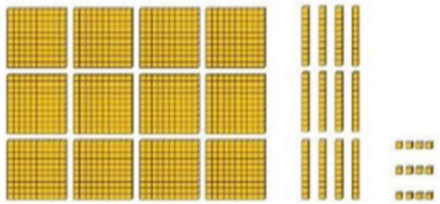
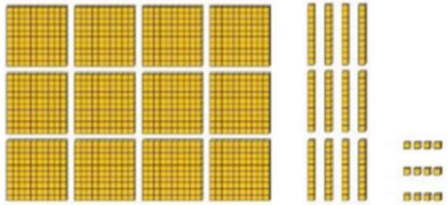
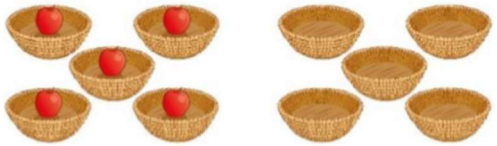
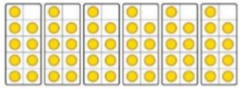
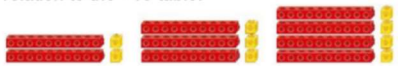
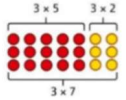
Multiplication

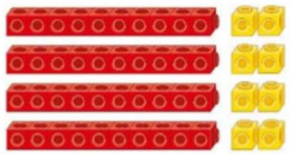
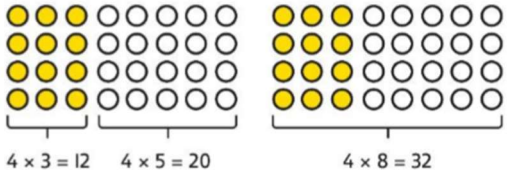
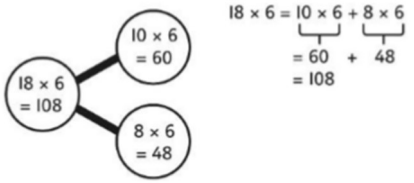
Key language: double, times, multiplied by, the product of, groups of, lots of, equal groups. Arrays, factor, multiple, commutative,

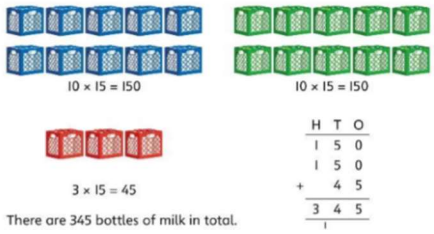
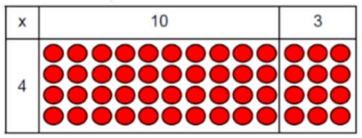
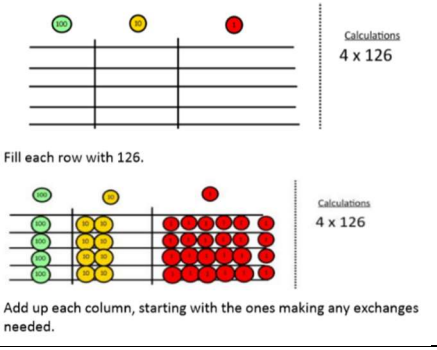
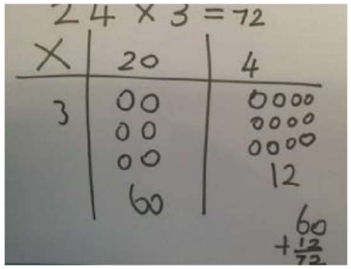
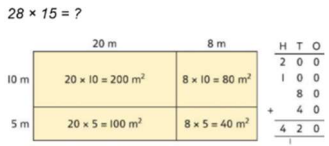
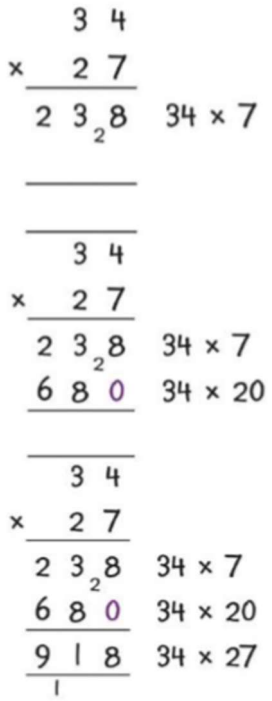
Objective	Concrete	Pictorial	Abstract
Doubling	Use practical activities to show how to double a number.  	Draw pictures to show how to double a number.  	Partition a number and then double each part before recombining it back together. 
Recognising and making equal groups	Children arrange objects in equal and unequal groups and understand how to recognise whether they are equal.  	Children draw and represent equal and unequal groups.  	Describe equal groups using words Three equal groups of 4. Four equal groups of 3.
Counting multiples	There are 5 pens in each pack … 5…10…15…20…25…30…35…40… 	100 squares and ten frames support counting in 2s, 5s and 10s. 	Use a number line to support repeated addition through counting in 2s, 5s and 10s. 

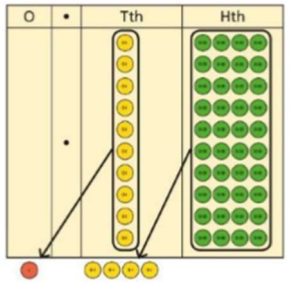
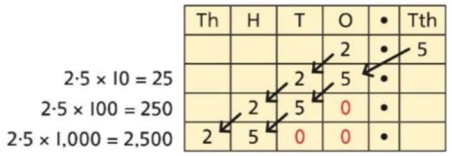
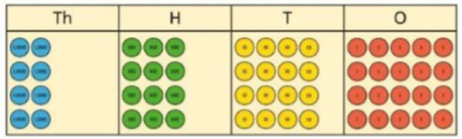
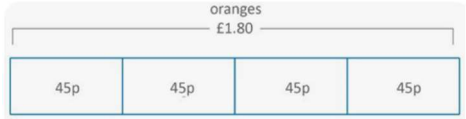
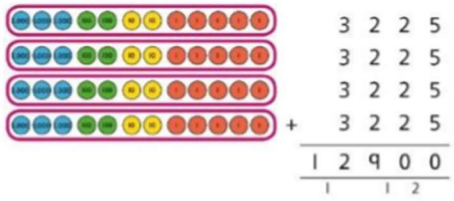
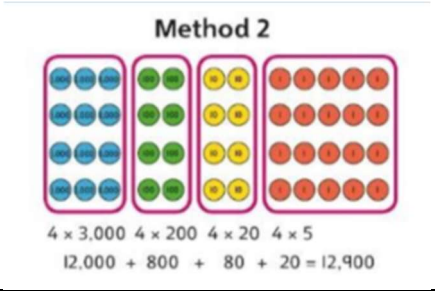
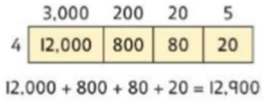
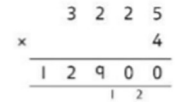
Equal groups / Repeated addition	 *3 groups of 5 chairs* *15 chairs altogether*	 *3 groups of 5* *15 in total*	 *5 + 5 + 5 = 15* *3 × 5 = 15*
Arrays – showing commutative multiplication	Create arrays using counters/ cubes to show multiplication sentences.  *This is 2 groups of 6 and also 6 groups of 2.*	Draw arrays in different rotations to find commutative multiplication sentences. 	Use arrays to make a range of number sentences. 4 + 4 + 4 + 4 + 4 = 20 5 + 5 + 5 + 5 = 20 4 × 5 = 20 and 5 × 4 = 20
Learning x2, x5 and x10 table facts	Develop an understanding of how to unitise groups of 2, 5 and 10 and learn corresponding times-table facts. 	Understand how to relate counting in unitised groups and repeated addition with knowing key times-table facts.  *10 + 10 + 10 = 30* *3 × 10 = 30*	Understand how the times-tables increase and contain patterns. 

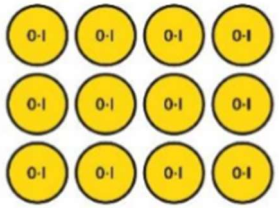
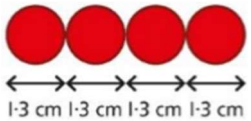
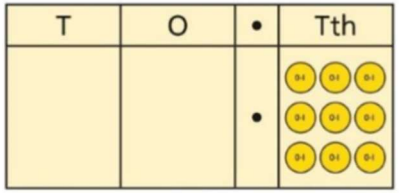
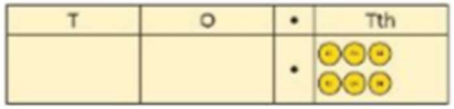
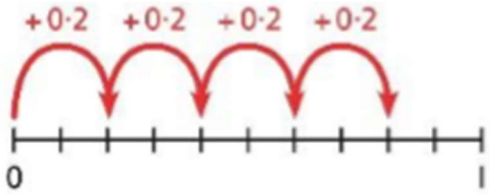
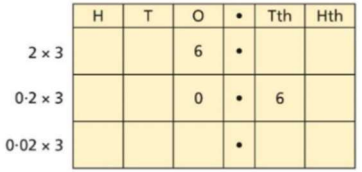
Using known facts to multiply 10s, for example 3 × 40	Explore the relationship between known timestables and multiples of 10 using place value equipment. Make 4 groups of 3 ones.  Make 4 groups of 3 tens.  What is the same? What is different?	Understand how unitising 10s supports multiplying by multiples of 10.  4 groups of 2 ones is 8 ones. 4 groups of 2 tens is 8 tens. 4 × 2 = 8 4 × 20 = 80	Understand how to use known times-tables to multiply multiples of 10.  4 × 2 = 8 4 × 20 = 80
Multiplying a single digit by a 2-digit number	Understand how to link partitioning a 2-digit number with multiplying. Each person has 23 flowers. Each person has 2 tens and 3 ones.  There are 3 groups of 2 tens. There are 3 groups of 3 ones. Use place value to support how partitioning is linked with multiplying by a 2-digit number.  	 *3 × 4 = 12*  *3 × 20 = 60* *60 + 12 = 72* *3 × 24 = 72*	Use addition to complete multiplications of 2digit numbers by a 1-digit number. 4 × 13 = ? 4 × 3 = 12 and 4 × 10 = 40 12 + 40 = 52 4 × 13 = 52

Multiply by 10 and 100	Use unitising and place value equipment to understand how to multiply by multiples of 1, 10 and 100.  3 groups of 4 ones is 12 ones. 3 groups of 4 tens is 12 tens. 3 groups of 4 hundreds is 12 hundreds	Use unitising and place value equipment to understand how to multiply by multiples of 1, 10 and 100.  3 × 4 = 12 3 × 40 = 120 3 × 400 = 1,200	Use known facts and understanding of place value and commutativity to multiply mentally. 4 × 7 = 28 4 × 70 = 280 40 × 7 = 280 4 × 700 = 2,800 400 × 7 = 2,800
Understand times tables up to 12 x 12	Understand the special cases of multiplying by 1 and 0.  *5 × 1 = 5* *5 × 0 = 0*	Represent the relationship between the ×9 table and the ×10 table.  Represent the ×11 table and ×12 tables in relation to the ×10 table.  *2 × 11 = 20 + 2* *3 × 11 = 30 + 3* *4 × 11 = 40 + 4*  *4 × 12 = 40 + 8*	Understand how times-tables relate to counting patterns. Understand links between the ×3 table, ×6 table and ×9 table 5 × 6 is double 5 × 3 ×5 table and ×6 table I know that 7 × 5 = 35 so I know that 7 × 6 = 35 + 7. ×5 table and ×7 table 3 × 7 = 3 × 5 + 3 × 2  ×9 table and ×10 table 6 × 10 = 60 6 × 9 = 60 − 6

Use partitioning in multiplication	Make multiplications by partitioning.  4 × 12 is 4 groups of 10 and 4 groups of 2.	Understand how multiplication and partitioning are related through addition.  4 × 3 = 12 4 × 5 = 20 12 + 20 = 32 4 × 8 = 32	Use partitioning to multiply 2-digit numbers by a single digit. 18 × 6 = ?  18 × 6 = 10 × 6 + 8 × 6 = 60 + 48 = 108
Multiplying more than 2 numbers	Represent situations by multiplying three numbers together.  Each sheet has 2 × 5 stickers. There are 3 sheets. There are 5 × 2 × 3 stickers in total. $5 \times 2 \times 3 = 30$ $10 \times 3 = 30$	Understand that commutativity can be used to multiply in different orders.  2 × 6 × 10 = 120 12 × 10 = 120 10 × 6 × 2 = 120 60 × 2 = 120	Use knowledge of factors to simplify some multiplications. $24 \times 5 = 12 \times 2 \times 5$ $12 \times 2 \times 5 =$ $12 \times 10 = 120$ So, $24 \times 5 = 120$

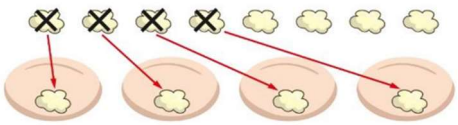
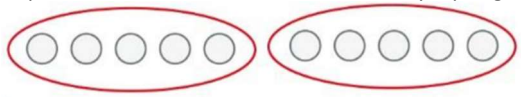
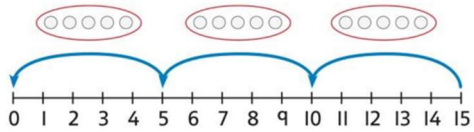
To multiply 2- and 3-digit numbers.	Partition one number into 10s and 1s, then add the parts. *23 × 15 = ?*  Show the link with arrays to first introduce the grid method.  Move on to place value counters to show how we are finding groups of a number.We are multiplying by 4 so we need 4 rows.  	Use an area model and add the parts. They can draw the counters, using colours to show different amounts or just use circles in the different columns to show their thinking as shown below.  	Use an area model using digits.  Use column multiplication, ensuring understanding of place value at each stage. 

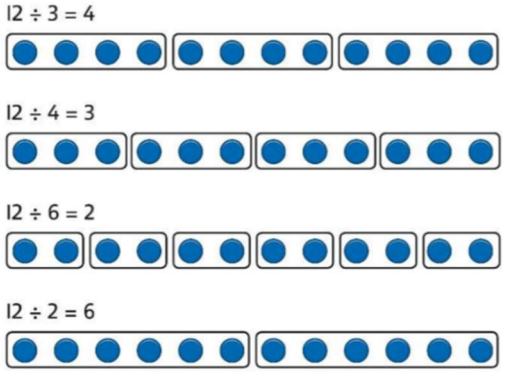
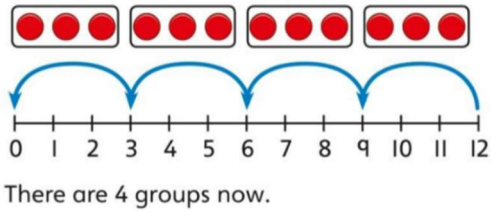
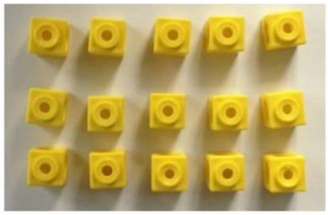
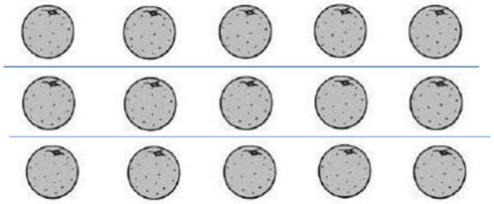
Multiplying decimals by 10, 100 and 1000.	Use place value equipment to explore and understand the exchange of 10 tenths, 10 hundredths or 10 thousandths.	Represent multiplication by 10 as exchange on a place value grid.  0·14 × 10 = 1·4	Understand how this exchange is represented on a place value chart. 
Column multiplication	Use equipment to explore multiplications.  4 groups of 2,345	Bar modelling and number lines can support learners when solving problems with multiplication alongside the formal written methods.  Use place value equipment to compare methods. <u>Method 1</u>  <u>Method 2</u>  	Understand area model and short multiplication. Compare and select appropriate methods for specific multiplications. <u>Method 3 – area model</u>  12,000 + 800 + 80 + 20 = 12,900 <u>Method 4 – short multiplication</u>  <u>Method 5 – long multiplication</u> 1 2 4 <u>x 2 6</u> 7 4 4 <u>2 4 8 0</u> 3 2 2 4

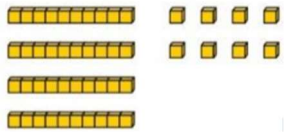
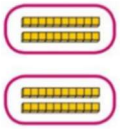
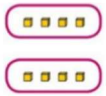
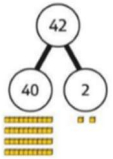
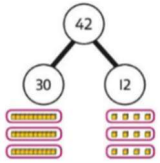
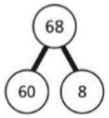
Multiplying decimals	Explore decimal multiplications using place value equipment and in the context of measures.  3 groups of 4 tenths is 12 tenths. 4 groups of 3 tenths is 12 tenths.  4 × 1 cm = 4 cm 4 × 0·3 cm = 1.2 cm 4 × 1·3 = 4 + 1·2 = 5·2 cm	Represent calculations on a place value grid. 3 × 3 = 9 3 × 0·3 = 0·9  Understand the link between multiplying decimals and repeated addition.   	Find families of facts from a known multiplication. I know that 18 × 4 = 72. This can help me work out: 1·8 × 4 = ? 18 × 0·4 = ? 180 × 0·4 = ? 18 × 0·04 = ? Use a place value grid to understand the effects of multiplying decimals. 

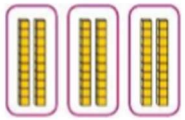
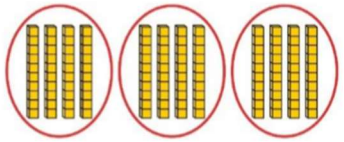
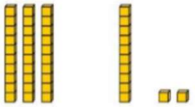
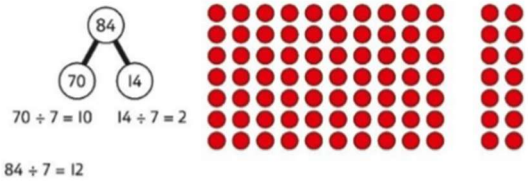
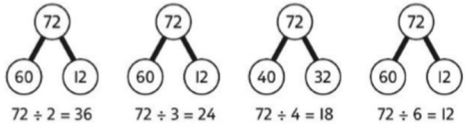
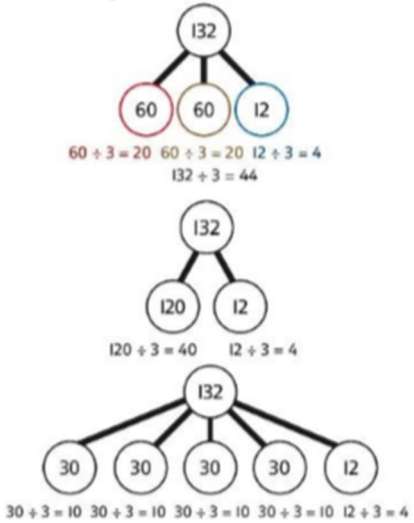
Division

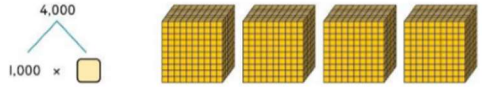
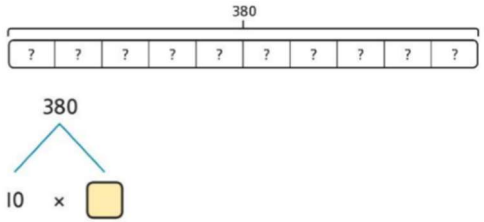
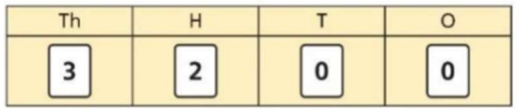
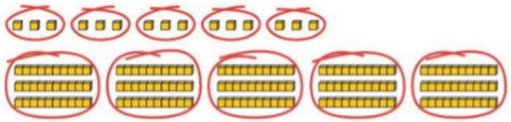
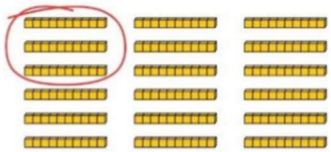
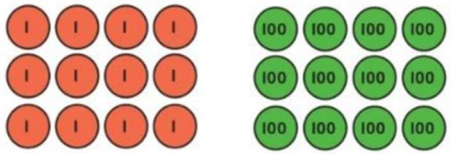
Key language: share, group, divide, divided by, half. equal, dividend, divisor, quotient, factor, multiple, remainder

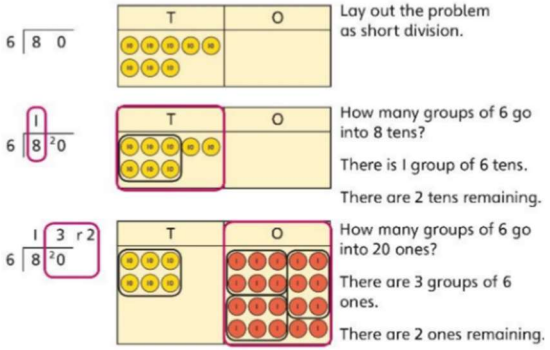
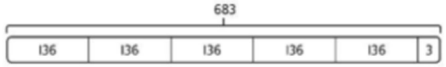
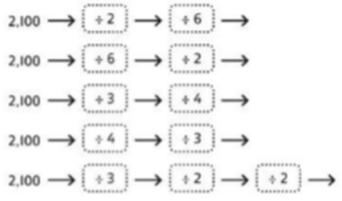
Objective	Concrete	Pictorial	Abstract
Sharing objects	Grouping Learn to make equal groups from a whole and find how many equal groups of a certain size can be made. Sort a whole set people and objects into equal groups.  *There are 10 children altogether.* *There are 2 in each group.* *There are 5 groups.* Sharing Share a set of objects into equal parts and work out how many are in each part. 	Grouping Represent a whole and work out how many equal groups.  There are 10 in total. There are 5 in each group. There are 2 equal groups. Sharing Sketch or draw to represent sharing into equal parts. This may be related to fractions. 	Grouping Children may relate this to counting back in steps of 2, 5 or 10.  Sharing 10 shared into 2 equal groups gives 5 in each group. **Share 9 buns between three people.** **9 ÷ 3 = 3**

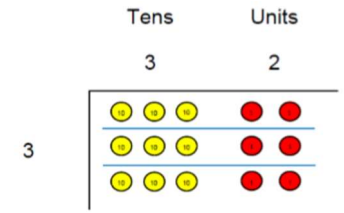
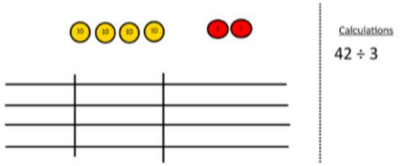
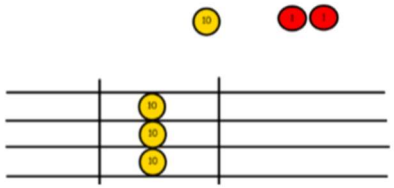
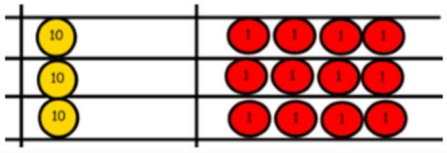
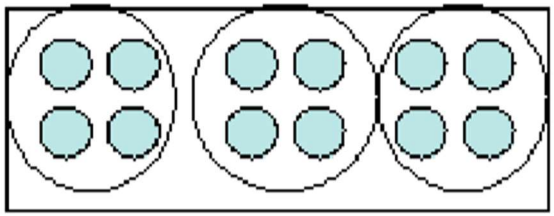
Division as grouping	Understand how to make equal groups from a whole.  8 divided into 4 equal groups. There are 2 in each group.	Understand the relationship between grouping and the division statements. 	Understand how to relate division by grouping to repeated subtraction.  12 divided into groups of 3. 12 ÷ 3 = 4 There are 4 groups.
Division within arrays	Link division to multiplication by creating an array and thinking about the number sentences that can be created.  E.g. 15 ÷ 3 = 5 5 x 3 = 15 15 ÷ 5 = 3 3 x 5 = 15	Draw an array and use lines to split the array into groups to make multiplication and division sentences. 	Find the inverse of multiplication and division sentences by creating four linking number sentences. 7 x 4 = 28 4 x 7 = 28 28 ÷ 7 = 4 28 ÷ 4 = 7

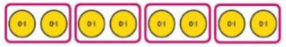
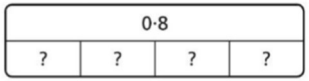
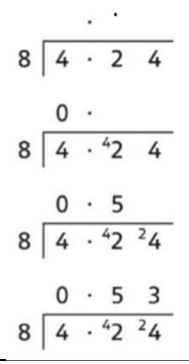
2-digit by 1-digit division without remainders	Children explore dividing 2-digit numbers by using place value equipment.  48 ÷ 2 = ? First divide the 10s.  Then divide the 1s 	Children explore which partitions support particular divisions.  I need to partition 42 differently to divide by 3.  42 = 30 + 12 42 ÷ 3 = 14	Children partition a number into 10s and 1s to divide where appropriate.  60 ÷ 2 = 30 8 ÷ 2 = 4 30 + 4 = 34 68 ÷ 2 = 34 Children partition flexibly to divide where appropriate. 42 ÷ 3 = ? 42 = 40 + 2 I need to partition 42 differently to divide by 3. 42 = 30 + 12 30 ÷ 3 = 10 12 ÷ 3 = 4 10 + 4 = 14 42 ÷ 3 = 14
2-digit number by 1-digit number division with remainders	Use place value equipment to understand the concept of remainder. Make 29 from place value equipment. Share it into 2 equal groups.  *There are two groups of 14 and 1 remainder.*	Use place value equipment to understand the concept of remainder in division. 29 ÷ 2 = ?  *29 ÷ 2 = 14 remainder 1*	Partition to divide, understanding the remainder in context. 67 children try to make 5 equal lines. 67 = 50 + 17 50 ÷ 5 = 10 17 ÷ 5 = 3 remainder 2 67 ÷ 5 = 13 remainder 2 There are 13 children in each line and 2 children left out.

Use known facts to divide multiples of 10	Use place value equipment to understand how to divide by unitising. Make 6 ones divided by 3.  Now make 6 tens divided by 3.  What is the same? What is different?	Divide multiples of 10 by unitising.  12 tens shared into 3 equal groups. 4 tens in each group.	Divide multiples of 10 by a single digit using known times-tables. 180 ÷ 3 = ? 180 is 18 tens. 18 divided by 3 is 6. 18 tens divided by 3 is 6 tens
Dividing 2- and 3-digit numbers by a single digit, using flexible partitioning	Use place value equipment to explore why different partitions are needed. 42 ÷ 3 = ? I will split it into 30 and 12, so that I can divide by 3 more easily. 	Represent how to partition flexibly where needed. 84 ÷ 7 = ? I will partition into 70 and 14 because I am dividing by 7. 	Make decisions about appropriate partitioning based on the division required.  Understand that different partitions can be used to complete the same division. 

Divide by 10, 100, 1000	Use place value equipment to support unitising for division. 4,000 ÷ 1,000  4,000 is 4 thousands. 4 × 1,000= 4,000 So, 4,000 ÷ 1,000 = 4	Use a bar model to support dividing by unitising. 380 ÷ 10 = 38  380 is 38 tens. 38 × 10 = 380 10 × 38 = 380 So, 380 ÷ 10 = 38	Understand how and why the digits change on a place value grid when dividing by 10, 100 or 1,000.  3,200 ÷ 100 = ? 3,200 is 3 thousands and 2 hundreds. 200 ÷ 100 = 2 3,000 ÷ 100 = 30 3,200 ÷ 100 = 32 So, the digits will move two places to the right.
Divide by a multiple of 10	Use place value equipment to represent known facts and unitising.  15 ones put into groups of 3 ones. There are 5 groups. 15 ÷ 3 = 5 15 tens put into groups of 3 tens. There are 5 groups. 150 ÷ 30 = 5	Represent related facts with place value equipment when dividing by unitising.  180 is 18 tens. 18 tens divided into groups of 3 tens. There are 6 groups. 180 ÷ 30 = 6  12 ones divided into groups of 4. There are 3 groups. 12 hundreds divided into groups of 4 hundreds. There are 3 groups. 1200 ÷ 400 = 3	Reason from known facts, based on understanding of unitising. Use knowledge of the inverse relationship to check. 3,000 ÷ 5 = 600 3,000 ÷ 50 = 60 3,000 ÷ 500 = 6 5 × 600 = 3,000 50 × 60 = 3,000 500 × 6 = 3,000

Understanding the remainder	Understand remainders using concrete versions of a problem. 80 cakes divided into trays of 6.  80 cakes in total. They make 13 groups of 6, with 2 remaining.	Use short division and understand remainders as the last remaining 1s. 	In problem solving contexts, represent divisions including remainders with a bar model.  683 = 136 × 5 + 3 683 ÷ 5 = 136 r 3
Understanding the relationship between division and fractions	Use sharing to explore the link between fractions and division. 1 whole shared between 3 people. Each person receives one-third. 	Use a bar model and other fraction representations to show the link between fractions and division.  $1 \div 3 = \frac{1}{3}$	Use the link between division and fractions to calculate divisions. $5 \div 4 = \frac{5}{4} = 1\frac{1}{4}$ $11 \div 4 = \frac{11}{4} = 2\frac{3}{4}$
Division by a 2-digit number using factors	Understand that division by factors can be used when dividing by a number that is not prime.	Use factors and repeated division. 1,260 ÷ 14 = ?  1,260 ÷ 2 = 630 630 ÷ 7 = 90 1,260 ÷ 14 = 90	Use factors and repeated division where appropriate. 2,100 ÷ 12 = ? 

Short and long division	Use place value counters to divide using the bus stop method alongside  Start with the biggest place value, we are sharing 40 into three groups. We can put 1 ten in each group and we have 1 ten left over.  We exchange this ten for ten ones and then share the ones equally among the groups.  We look how much in 1 group so the answer is 14. 	Students can continue to use drawn diagrams with dots or circles to help them divide numbers into equal groups.  Encourage them to move towards counting in multiples to divide more efficiently.	Begin with divisions that divide equally with no remainder. $\begin{array}{r} 2\ 1\ 8 \\ 4\overline{)\ 8\ 7\ 2} \\ \quad\ 3 \end{array}$ Move onto divisions with a remainder. $\begin{array}{r} 8\ 6\ \text{r}\ 2 \\ 5\overline{)\ 4\ 3\ 2} \\ \quad\ 3 \end{array}$ Use long division when working with 2- or 3-digit numbers. $\begin{array}{r} 0\ 1\ 5 \\ 31\overline{)\ 4\ 6\ 5} \\ \underline{\ 3\ 1\ \ } \\ \quad 1\ 5\ 5 \\ \quad \underline{1\ 5\ 5} \\ \quad\quad\ 0 \end{array}$ Progress onto short division $\begin{array}{r} 0\ 1\ 5 \\ 31\overline{)\ 4\ {}^{4}6\ {}^{15}5} \end{array}$

Dividing decimals by 10, 100 and 1000.	Use place value equipment to explore division of decimals.  8 tenths divided into 4 groups. 2 tenths in each group.	Use a bar model to represent divisions.  $4 \times 2 = 8$ $8 \div 4 = 2$ So, $4 \times 0{\cdot}2 = 0{\cdot}8$ $0{\cdot}8 \div 4 = 0{\cdot}2$	Use short division to divide decimals with up to 2 decimal places. 

Appendix 1: Example Questions for Mental Strategies (KS1)

Addition

	Example Question	Possible Counting Strategy
Year 1	What is the number between 4 and 6?	One – one correspondence
	13 + 5	Counting on
	3 + 4	Adding near doubles
		Count and add sets of objects together.
Year 2	24 + 6	Counting on from a value on a 100 square.
	4 + 3 + 7	Reordering calculations
	23 + 5	Counting on from 2-digit numbers
	36 + 5	Creating number bonds to 10
	27 + 60	Count on in 10's
	13 + 14	Add near doubles
	14 + 9 14 + 11	Add near 10 values

Subtraction

	Example Question	Possible Counting Strategy
Year 1	8 – 3	Counting back
	15 – 3	2-digit by 1-digit counting back
	10 – 4	Subtracting from 10
Year 2	12 – 7	Bridging 10
	30 – 5	Subtract from multiple of 10
	44 – 41	Finding the difference
	62 – 30	Subtracting in multiples of 10
	90 – 27	Partitioning / counting back in tens and ones
	27 – 9 28 – 11	Take away close multiples to 10
	86 – 23	2-digit by 2-digit subtraction without bridging the ten

Multiplication

	Example Question	Possible Counting Strategy
Year 1	2, 4, 6, __	Repeated addition
	5, 10, __	
	10, 20, __	
Year 2	20 x 2	Double multiples of 10
	6 x 5	Secure 2's, 5's and 10's times tables
	4 x 2 / 2 x 4	Commutativity
	10 + 10 + 10 + 10 / 10 x 4	Repeated addition

Division

	Example Question	Possible Counting Strategy
Year 1	10, 8, 6, …	Repeated subtraction to 0
	10, 5, …	
	20, 10, …	
Year 2	40 ÷ 2	Use knowledge of base 10 number system (4 ÷ 2)
	36 ÷ 2	Partition values in various ways
	30 ÷ 5 60 ÷ 10 16 ÷ 2	Use knowledge of 2, 5 and 10's times table.

Appendix 2 – Example Questions for Mental Strategies (KS2) (Addition and Subtraction)

Counting Forwards and Backwards

Children first meet counting by beginning at one and counting on in ones. Their sense of number is extended by beginning at different numbers and counting forwards and backwards in steps, not only of ones, but also of twos, fives, tens, hundreds, tenths and so on. The image of a number line helps them to appreciate the idea of counting forwards and backwards. They will also learn that, when they add two numbers together, it is generally easier to count on from the larger number rather than the smaller. You will need to review children's 'counting on' strategies, then show them and encourage them to adopt more efficient methods.

	Example Questions	Possible Counting Strategies
YEAR 3	50 + 38	count on in tens then ones from 50
	90 – 27	count back in tens then ones from 90
	34 + 65	count on in tens then ones from 34
	87 – 23	count back in tens then ones from 87
	35 + 15	count on in steps of 5 from 35
YEAR 4	73 – 68	count up from 68, counting 2 to 70 then 3 to 73
	47 + 58	count on 50 from 47, then 3 to 100, then 5 to 105
	124 – 47	count back 40 from 124, then 4 to 80, then 3 to 77
	570 + 300	count on in hundreds from 570
	960 – 500	count back in hundreds from 960
YEAR 5	3.2 + 0.6	count on in tenths
YEAR 6	1.7 + 0.55	count on in tenths and hundredths

Reordering

Sometimes a calculation can be more easily worked out by changing the order of the numbers. The way in which children rearrange numbers in a particular calculation will depend on which number facts they can recall or derive quickly.

It is important for children to know when numbers can be reordered:

e.g. 2 + 5 + 8 = 8 + 2 + 5 or 15 + 8 – 5 = 15 – 5 + 8 or 23 – 9 – 3 = 23 – 3 – 9

and when they can't be reordered:

e.g. 8 – 5 ≠ 5 – 8

The strategy of changing the order of numbers applies mainly when the question is written down. It is more difficult to reorder numbers if the question is presented orally.

	Example Questions	Possible Counting Strategies
YEAR 3	23 + 54	54 + 23
	12 – 7 – 2	12 – 2 – 7
	13 + 21 + 13	13 + 13 + 21 (using double 13)
YEAR 4	6 + 13 + 4 + 3	6 + 4 + 13 + 3
	17 + 9 – 7	17 – 7 + 9
	28 + 75	75 + 28 (thinking of 28 as 25 + 3)
YEAR 5	12 + 17 + 8 + 3	12 + 8 + 17 + 3
	25 + 36 + 75	25 + 75 + 36
	58 + 47 – 38	58 – 38 + 47
	200 + 567	567 + 200
	1.7 + 2.8 + 0.3	1.7 + 0.3 + 2.8
YEAR 6	3 + 8 + 7 + 6 + 2	3 + 7 + 8 + 2 + 6
	34 + 27 + 46	34 + 46 + 27
	180 + 650	650 + 180 (thinking of 180 as 150 + 30)
	1.7 + 2.8 + 0.3	1.7 + 0.3 + 2.8
	4.7 + 5.6 – 0.7	4.7 – 0.7 + 5.6 = 4 + 5.6

Partitioning: Counting on or back

It is important for children to know that numbers can be partitioned into, for example, hundreds, tens and ones, so that 326 = 300 + 20 + 6. In this way, numbers are seen as wholes, rather than as a collection of single digits in columns.

This way of partitioning numbers can be a useful strategy for adding and subtracting pairs of numbers. Both numbers can be partitioned, although it is often helpful to keep the first number as it is and to partition just the second number.

	Example Questions	Possible Counting Strategies
YEAR 3	23 + 45	40 + 5 + 20 + 3 = 40 + 20 + 5 + 3
	68 – 32	60 + 8 – 30 – 2 = 60 – 30 + 8 – 2
YEAR 4	55 + 37	55 + 30 + 7 = 85 + 7
	365 – 40	300 + 60 + 5 – 40 = 300 + 60 – 40 + 5
YEAR 5	43 + 28 + 51	40 + 3 + 20 + 8 + 50 + 1 = 40 + 20 + 50 + 3 + 8 + 1
	5.6 + 3.7	5.6 + 3 + 0.7 = 8.6 + 0.7
	4.7 – 3.5	4.7 – 3 – 0.5
YEAR 6	540 + 280	540 + 200 + 80
	276 – 153	276 – 100 – 50 – 3

Partitioning: Bridging through multiples of 10

An important aspect of having an appreciation of number is to know how close a number is to the next or the previous multiple of 10: to recognise, for example, that 47 is 3 away from 50, or that 47 is 7 away from 40. In mental addition or subtraction, it is often useful to count on or back in two steps, bridging a multiple of 10. The empty number line, with multiples of 10 as 'landmarks', is helpful, since children can visualise jumping to them. For example, 6 + 7 is worked out in two jumps, first to 10, then to 13.

Subtraction, the inverse of addition, can be worked out by counting back from the larger number. But it can also be represented as the difference or 'distance' between two numbers. The distance is often found by counting up from the smaller to the larger number, again bridging through multiples of 10 or 100. This method of complementary addition is sometimes called 'shopkeeper's method' because it is like a shop assistant counting out change. So the change from £1 for a purchase of 37p is found by counting coins into the hand: '37p and 3p is 40p, and 10p makes 50p, and 50p makes £1'.

	Example Questions	Possible Counting Strategies
YEAR 3	49 + 32	49 + 1 + 31
	90 – 27	27 + 3 + 60
YEAR 4	57 + 34 or 92 – 25	57 + 3 + 31 or 92 – 2 – 20 – 3
	84 – 35	35 + 5 + 40 + 4
YEAR 5	607 – 288	288 + 12 + 300 + 7
	6070 – 4987	4987 + 13 + 1000 + 70
YEAR 6	1.4 + 1.7 or 5.6 – 3.7	1.4 + 0.6 + 1.1 or 5.6 – 0.6 – 3 – 0.1
	0.8 + 0.35	0.8 + 0.2 + 0.15
	8.3 – 2.8	2.8 + 0.2 + 5.3 or 8.3 – 2.3 – 0.5

Partitioning: Compensating

This strategy is useful for adding and subtracting numbers that are close to a multiple of 10, such as numbers that end in 1 or 2, or 8 or 9. The number to be added or subtracted is rounded to a multiple of 10 plus or minus a small number. For example, adding 9 is carried out by adding 10, then subtracting 1; subtracting 18 is carried out by subtracting 20, then adding 2. A similar strategy works for adding or subtracting decimals that are close to whole numbers.
For example: 1.4 + 2.9 = 1.4 + 3 – 0.1 or 2.45 – 1.9 = 2.45 – 2 + 0.1.

	Example Questions	Possible Counting Strategies
YEAR 3	53 + 12	53 + 10 + 2
	53 – 12	53 – 10 – 2
	53 + 18	53 + 20 – 2
	84 – 18	84 – 20 + 2
YEAR 4	38 + 68	38 + 70 – 2
	95 – 78	95 – 80 + 2
	58 + 32	58 + 30 + 2
	64 – 32	64 – 30 – 2
YEAR 5	138 + 69	138 + 70 – 1
	405 – 399	405 – 400 + 1
YEAR 6	2½ + 1¾	2½ + 2 – ¼
	5.7 + 3.9	5.7 + 4.0 – 0.1
	6.8 – 4.9	6.8 – 5.0 + 0.1

Partitioning: Using near doubles

If children have instant recall of doubles, they can use this information when adding two numbers that are very close to each other. So, knowing that 6 + 6 = 12, they can be encouraged to use this to help them find 7 + 6, rather than use a counting on strategy or bridging through 10.

	Example Questions	Possible Counting Strategies
YEAR 3	18 + 16	is double 18 and subtract 2 or double 16 and add 2
	60 + 70	is double 60 and add 10 or double 70 and subtract 1
YEAR 4	76 + 75	is double 76 and subtract 1 or double 75 and add 1
YEAR 5	160 + 170	is double 150, then add 10, then add 20 or double 160 and add 10 or double 170 and subtract 10
YEAR 6	2.5 + 2.6	is double 2.5 and add 0.1 or double 2.6 and subtract 0.1

Partitioning: Bridging through 60 to calculate a time interval

Time is a universal non-metric measure.
A digital clock displaying 9.59 will, in two minutes time, read 10.01 not 9.61. When children use minutes and hours to calculate time intervals, they have to bridge through 60.
So to find the time 20 minutes after 8.50am, for example, children might say 8.50am plus 10 minutes takes us to 9.00am, then add another 10 minute.

	Examples of Mental Questions
YEAR 3	It is 10.30am. How many minutes to 10.45am?
	It is 3.45pm. How many minutes to 4.15pm?
YEAR 4	I get up 40 minutes after 6.30am. What time is that?
	What is the time 50 minutes before 1.10pm?
	It is 4.25pm. How many minutes to 5.05pm?
YEAR 5	What time will it be 26 minutes after 3.30am?
	What was the time 33 minutes before 2.15pm?
	It is 4.18pm. How many minutes to 5.00pm? 5.26pm?
YEAR 6	It is 08.35. How many minutes is it to 09.15?
	It is 11.45. How many hours and minutes is it to 15.20?
	A train leaves London for Leeds at 22.33. The journey takes 2 hours 47 minutes. What time does the train arrive?

Appendix 3 – Example Questions for Mental Strategies (KS2) (Multiplication and Division)

Multiplication and division facts up to 12 x 12

Fluent recall of multiplication and division facts relies on regular opportunities for practice. Generally, frequent short sessions are more effective than longer, less frequent sessions. It is crucial that the practice involves as wide a variety of activities, situations, questions and language as possible and that it leads to deriving and recognising number properties, such as doubles and halves, odd and even numbers, multiples, factors and primes.

	Example Questions
YEAR 3	Derive and recall doubles of multiples of 10 to 100 and corresponding halves
	Derive and recall multiplication facts for the 2, 3, 4, 5, 6 and 10 times-tables and corresponding division facts
	Recognise multiples of 2, 3, 4, 5, 6 and 10 up to the tenth multiple
YEAR 4	Identify doubles of two-digit numbers and corresponding halves
	Derive doubles of multiples of 10 and 100 and corresponding halves
	Derive and recall multiplication facts up to 10 × 10 and corresponding division facts
	Recognise multiples of 2, 3, 4, 5, 6, 7, 8, 9 and 10 up to the tenth multiple
YEAR 5	Recall squares of numbers to 10 × 10
	Use multiplication facts to derive products of pairs of multiples of 10 and 100 and corresponding division facts
YEAR 6	Recall squares of numbers to 12 × 12 and derive corresponding squares of multiples of 10
	Use place value and multiplication facts to derive related multiplication and division facts involving decimals (e.g. 0.8 × 7, 4.8 ÷ 6)
	Identify factor pairs of two-digit numbers
	Identify prime numbers less than 100

Doubling and halving

The ability to double numbers is useful for multiplication.
Historically, multiplication was carried out by a process of doubling and adding. Most people find doubles the easiest multiplication facts to remember, and they can be used to simplify other calculations.
Sometimes it can be helpful to halve one of the numbers in a multiplication calculation and double the other.

	Example Questions
YEAR 3	Double multiples of 10 to 100, e.g. double 90, and corresponding halves
	Double multiples of 5 to 100 and find the corresponding halves, e.g. double 85, halve 170
YEAR 4	Double any two-digit number and find the corresponding halves, e.g. double 47, half of 94
	Double multiples of 10 and 100 and find the corresponding halves, e.g. double 800, double 340, half of 1600, half of 680
YEAR 5	Form equivalent calculations and use doubling and halving, e.g. • multiply by 4 by doubling twice, e.g. 16 × 4 = 32 × 2 = 64 • multiply by 8 by doubling three times, e.g. 12 × 8 = 24 × 4 = 48 × 2 = 96 • divide by 4 by halving twice, e.g. 104 ÷ 4 = 52 ÷ 2 = 26 • divide by 8 by halving three times, e.g. 104 ÷ 8 = 52 ÷ 4 = 26 ÷ 2 = 13 • multiply by 5 by multiplying by 10 then halving, e.g. 18 × 5 = 180 ÷ 2 = 90 • multiply by 20 by doubling then multiplying by 10, e.g. 53 × 20 = 106 × 10 = 1060
	Multiply by 50 by multiplying by 100 and halving
	Multiply by 25 by multiplying by 100 and halving twice
YEAR 6	Double decimals with units and tenths, e.g. double 7.6, and find the corresponding halves, e.g. half of 15.2 Form equivalent calculations and use doubling and halving, e.g. • divide by 25 by dividing by 100 then multiplying by 4 e.g. 460 ÷ 25 = 4.6 × 4 = 18.4 • divide by 50 by dividing by 100 then doubling e.g. 270 ÷ 50 = 2.7 × 2 = 5.4

Multiplying and dividing by multiples of 10

Being able to multiply by 10 and multiples of 10 depends on an understanding of place value and knowledge of multiplication and division facts. This ability is fundamental to being able to multiply and divide larger numbers.

	Expectations with Example
YEAR 3	Multiply one-digit and two-digit numbers by 10 or 100, e.g. 7 × 100, 46 × 10, 54 × 100
	Change pounds to pence, e.g. £6 to 600 pence, £1.50 to 150 pence
YEAR 4	Multiply numbers to 1000 by 10 and then 100, e.g. 325 × 10, 42 × 100
	Divide numbers to 1000 by 10 and then 100 (whole-number answers), e.g. 120 ÷ 10, 600 ÷ 100, 850 ÷ 10
	Multiply a multiple of 10 to 100 by a single-digit number, e.g. 60 × 3, 50 × 7
	Change hours to minutes; convert between units involving multiples of 10 and 100, e.g. centimetres and millimetres, centilitres and millilitres, and convert between pounds and pence, metres and centimetres, e.g. 599 pence to £5.99, 2.5m to 250cm
YEAR 5	Multiply and divide whole numbers and decimals by 10, 100 or 1000, e.g. 4.3 × 10, 0.75 × 100, 25 ÷ 10, 673 ÷ 100
	Divide a multiple of 10 by a single-digit number (whole number answers), e.g. 80 ÷ 4, 270 ÷ 3
	Multiply pairs of multiples of 10, and a multiple of 100 by a single digit number, e.g. 60 × 30, 900 × 8
	Multiply by 25 or 50, e.g. 48 × 25, 32 × 50 using equivalent calculations, e.g. 48 x 100 ÷ 4, 32 x 100 ÷ 2
	Convert larger to smaller units of measurement using decimals to one place, e.g. change 2.6 kg to 2600 g, 3.5 cm to 35 mm, and 1.2 m to 120 cm
YEAR 6	Multiply pairs of multiples of 10 and 100, e.g. 50 × 30, 600 × 20
	Divide multiples of 100 by a multiple of 10 or 100 (whole number answers), e.g. 600 ÷ 20, 800 ÷ 400, 2100 ÷ 300
	Divide by 25 or 50
	Convert between units of measurement using decimals to two places, e.g. change 2.75 l to 2750 ml, or vice versa

Multiplying and dividing by single digit numbers and multiplying by two-digit numbers

Once children are familiar with some multiplication facts, they can extend their skills.

- One strategy is to partition one of the numbers and use the distributive law of multiplication over addition. So, for example, 6 × 7 = 6 × (5 + 2) = 6 × 5 + 6 × 2 or, in words, 'seven sixes are five sixes plus two sixes'. Subtraction can be used similarly, so 'nine eights are ten eights minus one eight'.
- Another strategy is to make use of factors, so 7 × 6 is seen as 7 × 3 × 2.

Once children understand the effect of multiplying and dividing by 10, they can start to extend their multiplication and division skills to larger numbers.

- A product such as 26 × 3 can be worked out by partitioning 26 into 20 + 6, multiplying each part by 3, then recombining.
- One strategy for multiplication by 2, 4, 8, 16, 32, … is to use doubling, so that 9 × 8 is seen as 9 × 2 × 2 × 2. A strategy for dividing by the same numbers is to use halving.
- A strategy for multiplying by 50 is to multiply by 100, then halve, and for multiplying by 25 is to multiply by 100 then divide by 4.

Since each of these strategies involves at least two steps, most children will find it helpful to make jottings of the intermediate steps in their calculations.

	Example Questions
YEAR 4	Find one quarter by halving one half
	Multiply numbers to 20 by a single-digit number, e.g. 17 × 3
YEAR 5	Multiply and divide two-digit numbers by 4 or 8, e.g. 26 × 4, 96 ÷ 8
	Multiply two-digit numbers by 5 or 20, e.g. 32 × 5, 14 × 20
	Multiply by 25 or 50, e.g. 48 × 25, 32 × 50
YEAR 6	Multiply a two-digit and a single-digit number, e.g. 28 × 7
	Divide a two-digit number by a single-digit number e.g. 68 ÷ 4
	Divide by 25 or 50, e.g. 480 × 25, 3200 × 50
	Find new facts from given facts, e.g. • given that three oranges cost 24p, find the cost of four oranges

Fractions, decimals and percentages.

Children need an understanding of how fractions, decimals and percentages relate to each other. For example, if they know that 1/2, 0.5 and 50% are all ways of representing the same part of a whole, then they can see that the calculations:

half of 40 ½ × 40 40 × ½ 40 × 0.5 0.5 × 40 50% of 40

These are different versions of the same calculation. Sometimes it might be easier to work with fractions, sometimes with decimals and sometimes with percentages.

There are strong links between this section and the earlier section 'Multiplying and dividing by multiples of 10'

	Example Questions
YEAR 3	Find half of any multiple of 10 up to 200, e.g. halve 170
	Find $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$ and $\frac{1}{10}$ of numbers in the 2, 3, 4, 5 and 10 times tables
YEAR 4	Find half of any even number to 200
	Find unit fractions and simple non-unit fractions of whole numbers or quantities, e.g. $\frac{3}{8}$ of 24
	Recall fraction and decimal equivalents for one-half, quarters, tenths and hundredths, e.g. recall the equivalence of 0.3 and $\frac{3}{10}$, and 0.03 and $\frac{3}{100}$
YEAR 5	Recall percentage equivalents of one-half, one-quarter, three-quarters, tenths and hundredths
	Find fractions of whole numbers or quantities, e.g. $\frac{2}{3}$ of 27, $\frac{4}{5}$ of 70 kg
	Find 50%, 25% or 10% of whole numbers or quantities, e.g. 25% of 20 kg, 10% of £80
YEAR 6	Recall equivalent fractions, decimals and percentages for hundredths, e.g. 35% is equivalent to 0.35 or $\frac{35}{100}$
	Find half of decimals with units and tenths, e.g. half of 3.2
	Find 10% or multiples of 10%, of whole numbers and quantities, e.g. 30% of 50 ml, 40% of £30, 70% of 200 g

Appendix 4: Teaching sequence of securing times tables

Under government guidance, children should be secure in their times tables up to 12 x 12 by the time they reach the end of year 4. To support this, Kensington primary school have implemented the use of TT Rockstars as an interactive learning platform that provides staff with guidance and next steps in their pupils' individual progress.

Suggested learning sequence

	Times tables
Year 2	10s, 2s, 5s,
Year 3	3s, 4s, 8s
Year 4	3s, 6s, 9s. 7s, 11s, 12s
Year 5 +	Securing gaps from previous years.

Week	Year 2/Grade 1	Year 3/Grade 2	Year 4/Grade 3	Year 5+/Grade 4+
Week 1	10s	3s	3s	3s
Week 2	10s	3s	6s	4s
Week 3	2s	4s	3s 6s	5s
Week 4	2s	4s	9s	3s 4s 5s
Week 5	2s 10s	8s	3s 9s	6s
Week 6	5s	3s 8s	3s 6s 9s	7s
Week 7	5s	4s	7s	6s 7s
Week 8	5s 10s	8s	7s	8s
Week 9	5s 10s	3s 4s 8s	2s 4s 8s	9s
Week 10	2s	3s 4s 8s	7s	10s
Week 11	2s 5s 10s	3s 4s 8s	6s 7s	8s 9s 10s
Week 12	2s 5s 10s	3s	9s	3s 4s 5s
Week 13	10s	3s	6s	6s 7s
Week 14	2s	4s	7s	8s 9s 10s
Week 15	5s	4s	6s 7s 9s	3s 4s 5s 6s 7s 8s 9s 10s

Week 16	2s 5s 10s	8s	6s 7s 8s 9s	7s 8s 9s 10s
Week 17	10s	3s 8s	11s	11s
Week 18	5s	4s	12s	12s
Week 19	5s 10s	3s 4s 5s 10s	11s 12s	6s 7s 8s 9s 10s 11s 12s
Week 20	2s 10s	2s 4s 8s	2s 3s 4s 5s 6s 7s 8s 9s	3s 4s 5s 6s 7s 8s 9s 10s 11s 12s
Week 21	2s 5s 10s	5s 8s 10s	6s 9s	6s
Week 22	5s 10s	2s 3s 4s 5s	7s 8s	7s
Week 23	2s 5s 10s	2s 3s 4s 5s 8s 10s	11s 12s	11s 12s
Week 24	2s 5s 10s	2s 3s 4s 5s 8s 10s	2s 3s 4s 5s 6s7s 8s 9s 10s 11s 12s	2s 3s 4s 5s 6s7s 8s 9s 10s 11s 12s

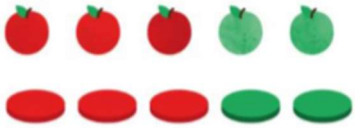
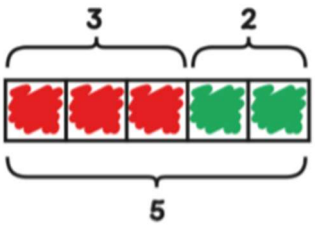
Motivation and Incentives

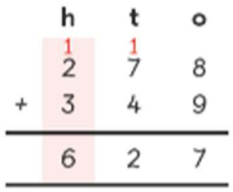
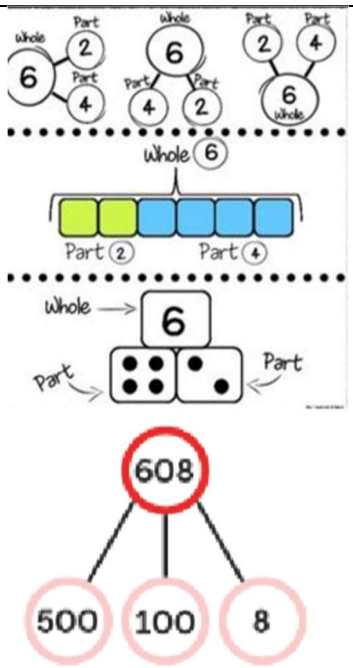
Regular whole-class checks are made to recognise progress and achievement on the learning platform. This should focus on effort and not just attainment. Children can be chosen for the weekly 'Mathemagician' certificate based on their improved / sustained use of the platform.

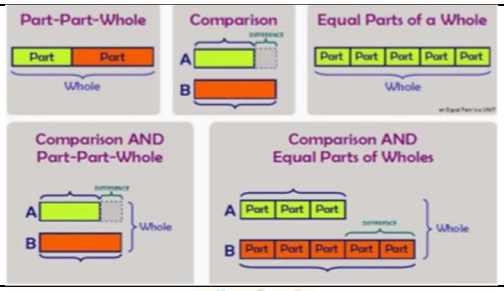
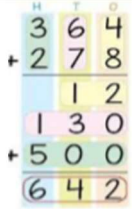
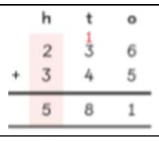
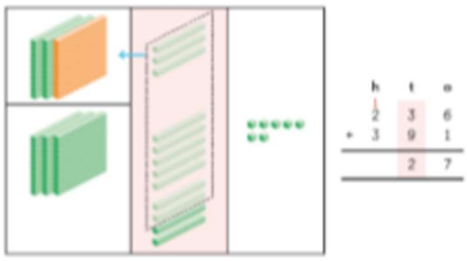
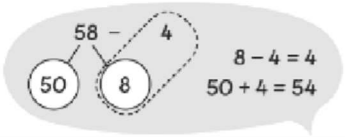
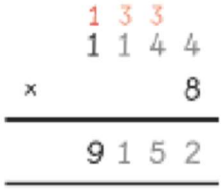
Competitions

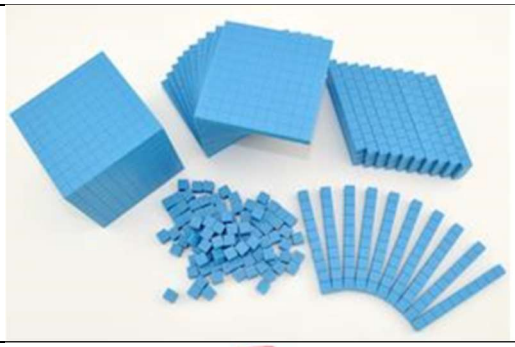
Internal Class Competitions: Classes can divide their pupils into groups to set tournaments over a period of a week / fortnight.
Year Group Competitions: Classes can challenge other classes in the year group to a tournament.
Whole School Competitions: Autumn and summer themed competitions take place between classes across the whole school, resulting in a class being awarded the TT Rockstar trophy.

Mastery Glossary

Term	Example	Meaning
Concrete	Any physical representation which the children can hold. e.g. fruit, flowers, cubes, base 10 etc 	Concrete is the "doing" stage, using concrete objects to model problems. Instead of the traditional method of maths teaching, where a teacher demonstrates how to solve a problem, the CPA approach brings concepts to life by allowing children to experience and handle physical objects themselves. Every new abstract concept is learned first with a "concrete" or physical experience. For example, if a problem is about adding up four baskets of fruit, the children might first handle actual fruit before progressing to handling counters or cubes which are used to represent the fruit
Pictorial	Any 2D drawing representation. e.g. bar model, story map etc.  	Pictorial is the "seeing" stage, using representations of the objects to model problems. This stage encourages children to make a mental connection between the physical object and abstract levels of understanding by drawing or looking at pictures, circles, diagrams or models which represent the objects in the problem. Building or drawing a model makes it easier for children to grasp concepts they traditionally find more difficult, such as fractions, as it helps them visualise the problem and make it more accessible.

Abstract	Any aspect where there is no involvement of pictorial or concrete representations including context e.g. column addition.  	Abstract is the "symbolic" stage, where children are able to use abstract symbols to model problems (Hauser) Only once a child has demonstrated that they have a solid understanding of the "concrete" and "pictorial" representations of the problem, can the teacher introduce the more "abstract" concept, such as mathematical symbols. Children are introduced to the concept at a symbolic level, using only numbers, notation, and mathematical symbols, for example +, –, x, / to indicate addition, multiplication, or division
Ones	Anything which represents 1 as a unit– this could be a cube, an apple, any number which is in the smallest whole number column etc.	Previously known as units - this is less abstract and represents that this is 1 whole that can be a part of a larger number.
Model		A representation (usually a diagram of some sort) which assists in understanding a problem without actually solving it.
Method		A series of steps with a particular order which will lead to finding the answer to a calculation
Calculation	e.g. 45 x 2 = 90	A symbolic representation of a problem.
Part-whole model	 	Any model which relates a whole to the sum of its parts. Most often represented as circles and relating to number bonds/partitioning. The part whole model is most often seen with two parts to one whole but it can be as many parts as needed to make the one whole.

Bar model		A way of representing a problem pictorially which helps to understand the different elements involved.
Column addition (expanded)		(expanded) Numbers are added together starting from the smallest column. Each column total is then lined up underneath to show the place value. These totals are then added together.
Column addition		Referring to the 'column' places focus on the place value of each digit. Both methods require the children to start at the lowest place value and move systematically from right to left.
Column subtraction		
Regrouping / exchanging		A term used to describe the process of changing groups of ones into tens to make adding and subtracting easier (in thousands into hundreds and so on). Although most common for add/sub regrouping can also be used when partitioning. Renaming/regrouping are often used interchangeably however regrouping would most often refer to the process and renaming the actual change in terms.
Partitioning / decomposing / number-bond	This is an example for subtraction but any method where partitioning the number and representing in a part-whole model is this method. 	Separating the original whole number into parts that are more useful when calculating especially when renaming is involved.
Short multiplication		Multiplying a single digit by any number, without expanding the addition – only one row as the answer. Expanded versions may be shown but these just link to partitioning rather than true long multiplication.

Long multiplication	1 3 2 × 1 2 2 6 4 → multiply by 2 + 1 3 2 0 → multiply by 10 1 5 8 4	Multiplying any number by more than one digit, leading to more than one row which needs to be totalled for a final answer
Short division	9 7 3) 2 9 2 1	Division algorithm in which the quotient is written directly without a succession of intermediate workings
Base 10		The physical representation of the decimal number system.
Cuisenaire rods		Any set of coloured rods/bars usually of 1-centimetre cross section and of 10 lengths from 1 to 10 centimetres that are used for teaching number concepts and the basic operations of arithmetic. Can be used to support bar modelling for example.